How do you model macroeconomic time series?

(Source: SMBC)

Econometrics: Computer Modelling

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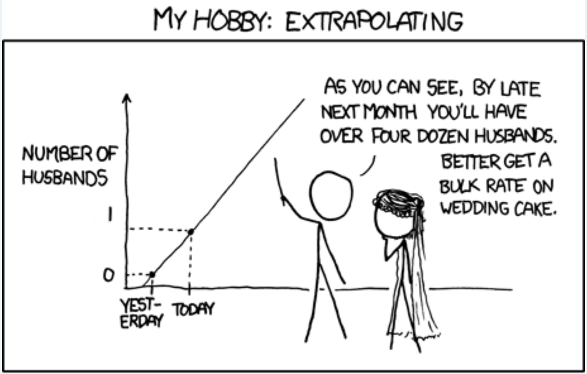
Lecture 3: Macro-Econometrics: Time Series

Structure

- 1: Intro to Econometric Software & Cross-Section Regression
- 2: Micro-Econometrics: Limited Indep. Variable
- 3: **Macro-Econometrics: Time Series**



Last time:

- Introduce econometric modelling in practice
- Introduce OxMetrics/PcGive Software
- Binary dependent variables & Count data

Today:

- Time Series
  - Dependence over time, dynamics, spurious relationships
- Hendry, D. F. (2015) *Introductory Macro-econometrics: A New Approach*.
  - Freely available online: `http://www.timberlake.co.uk/macroeconometrics.html`

- Economies high dimensional, interdependent, heterogeneous, and evolving: comprehensive specification of all events is impossible.
- Economic Theory
    - likely wrong and incomplete
    - **meaningless** without empirical support

    - Econometrics to discover new relationships from data
    - Econometrics can provide empirical support. . . or refutation.

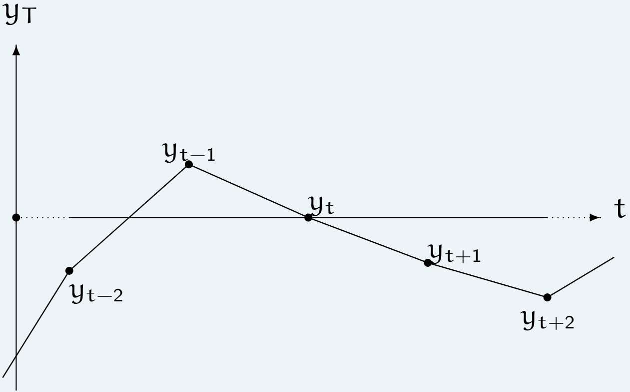
Structure of data

(Time Series – ordering)

	EmpUK	U	WPOP	POPUK	Ur
1860	11998.5	208.605	12207.1	missing	.0170888
1861	11884.3	426.279	12310.5	missing	.0346271
1862	11696.8	698.372	12395.2	missing	.0563422
1863	11936.	553.256	12489.2	missing	.0442986
1864	12347.1	226.744	12573.9	missing	.018033
1865	12450.2	217.674	12667.9	missing	.0171831
1866	12435.1	317.442	12752.6	missing	.0248924
1867	12084.7	761.86	12846.6	missing	.0593044
1868	12115.3	825.349	12940.7	missing	.0637795
1869	12299.7	725.581	13025.3	missing	.0557056
1870	12656.8	462.558	13119.3	missing	.0352577
1871	13013.9	199.535	13213.4	missing	.015101
1872	13189.5	117.907	13307.4	missing	.00886024
1873	13265.4	136.047	13401.5	30.0445	.0101516
1874	13286.9	208.605	13495.5	30.347	.0154573

- Realisation of a variable y at time t: y_t.
- Series $y_1, \ldots, y_T$: **time series**.
- Same data series at a number of (regular) periods in time.
  - E.g. GDP for UK, inflation, interest rates.
- Time series data distinguished by its **frequency**:
  - How often is the variable observed through time?
  - Yearly, quarterly, monthly, weekly, daily, hourly, by the minute?

Examples



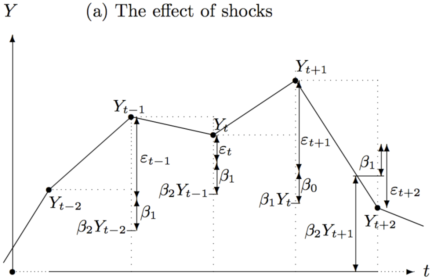
- Want to understand **persistence**:
  - Tells us much about economic variables.
    - E.g. price efficiency, partial adjustments, interest rate smoothing.
  - If we don't model it properly, can cause big mistakes.
- **Autoregressive models**:
  - Regression model of variable Y_t on itself in previous time period Y_{t-1}.
  - Additional common notation:
    - Lag operator: $L^k Y_t = Y_{t-k}$
    - Difference operator: $\Delta Y_t = (1-L)Y_t = Y_t - Y_{t-1}$
    - $\Delta^2 Y_t = (1-L)^2 Y_t = \Delta Y_t - \Delta Y_{t-1}$
    - $\Delta_2 Y_t = (1-L^2) Y_t = Y_t - Y_{t-2}$

- Autoregressive model has three elements:
  - **(1)** Where Y_t was the last time period.
  - **(2)** The unexpected event ϵ_t.
  - **(3)** Constant term allowing mean of Y_t to be non-zero.

$$Y_t = \underbrace{\alpha_0}_{(3)} + \underbrace{\alpha_1 Y_{t-1}}_{(1)} + \underbrace{\epsilon_t}_{(2)}, \quad \epsilon_t \sim \mathbb{N}[0, \sigma^2]. \tag{1}$$

Notation: normally use α for autoregressive, but equivalent to:

$$Y_t = \beta_1 + \beta_2 Y_{t-1} + \epsilon_t, \quad \epsilon_t \sim \mathbb{N}[0, \sigma^2]. \tag{2}$$



- AR(1) model allows us to determine many things about theory:
  - α_1: How quickly equilibrium re-established.
  - α_0 and α_1: Whether equilibrium is zero or otherwise.
  - σ^2: How much variation there is in Y_t around equilibrium.
    - How big are the unexpected events?
- What is equilibrium value? Taking expectations:

$$EY_t = \alpha_0 + \alpha_1 EY_{t-1}. \tag{3}$$

- Assume $EY_t = EY_{t-1}$ we find that $\mu_Y = EY = \alpha_0/(1-\alpha_1)$.
  - Define μ_Y as the equilibrium value, or **unconditional mean** of Y_t.

- We learn about the persistence of deviations from equilibrium from α_1.
- To see why note that $\mu_Y = \alpha_0/(1-\alpha_1)$ implies $\alpha_0 = \mu_Y(1-\alpha_1)$ so that:

$$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \epsilon_t \quad \Longrightarrow \quad Y_t - \mu_Y = \alpha_1(Y_{t-1} - \mu_Y) + \epsilon_t. \tag{4}$$

- We have **de-meaned** Y_t: We only care about α_1 and deviations from equilibrium.
- If assume **no more shocks happen** can see how quickly impact of shock disappears.
- $Y_t - \mu_Y = \alpha_1(Y_{t-1} - \mu_Y)$ and $Y_{t-1} - \mu_Y = \alpha_1(Y_{t-2} - \mu_Y)$ so:

$$Y_t - \mu_Y = \alpha_1^2(Y_{t-2} - \mu_Y). \tag{5}$$

- We can carry on doing this:

$$Y_t - \mu_Y = \alpha_1^3 (Y_{t-3} - \mu_Y) \quad \ldots \quad Y_t - \mu_Y = \alpha_1^k (Y_{t-k} - \mu_Y) \tag{6}$$

- It so happens that:

$$\text{Corr}\left[Y_t, Y_{t-k}\right] = \frac{Cov(Y_t, Y_{t-k})}{\sqrt{V(Y_t)}\sqrt{V(Y_{t-k})}} = \frac{\alpha_1^k \sigma_Y^2}{\sigma_Y \times \sigma_Y} = \alpha_1^k. \tag{7}$$

  - Measure autocorrelation (correlation through time) of Y_t from α_1!
- The higher is α_1 (nearer to 1) the more persistent is the series:
  - If $\alpha_1 = 0.9$ then $\alpha_1^2 = 0.81$ and $\alpha_1^{10} = 0.35$.
  - If $\alpha_1 = 0.2$ then $\alpha_1^2 = 0.04$ and $\alpha_1^{10} \approx 0$.

- May need more than one lag to explain dynamics of variable:
  - If we model p lags, we have AR(p) model.
- E.g. AR(2): $Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \epsilon_t$.
- Estimators like in **multivariate regression**:
  - $\hat{\alpha}_2$ asks Y_{t-1} to **be still**! It controls for first lag to get **only second lag effect**.

$$\hat{\alpha}_2 = \frac{\sum_{t=2}^{T} Y_{t-2}(Y_t | Y_{t-1})}{\sum_{t=2}^{T} Y_{t-2}(Y_{t-2} | Y_{t-1})}.$$

- Unconditional mean, variance and covariance affected. E.g. unconditional mean:

$$\mu_Y = \frac{\alpha_0}{1 - \alpha_1 - \alpha_2}. \tag{8}$$

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- Common method for learning about autocorrelation is graphically.
  - Autocorrelation function (ACF): $\mathrm{Corr}\,[Y_t, Y_{t-p}]$, $p = 1, 2, \ldots, 20$.
  - Partial ACF (PACF): $\mathrm{Corr}\,[Y_t, Y_{t-p} | Y_{t-1}, \ldots, Y_{t-p+1}]$, $p = 1, 2, \ldots, 20$.
    - 'Rule of Thumb': Number of significant PACF lags $\approx$ number of autoregressive lags needed in model.

Fulton Fish Market: Price, Quantity, Weather

- Load ”fish.in7”
- Series
  - Model for qty = log(Quantity)
  - Weather: Stormy, Rainy, Cold
- Graph the series! (Important first step!)
  - Time Series Plots

Construct Auto-regressive models for log(Quantity) sold:

- Determine lag length: Plot Partial Auto-correlation function (max 10 lags)
- Estimate an AR(1), AR(2) models
  - 'Models for Time Series Data'
  - 'Single Equation Dynamic Modelling'
  - `x_1` denotes the first lag of x, `x_2` the second, etc.
- What is the long-run equilibrium?
- Interpret mis-specification tests
  - Outlying observations?

```
EQ(15) Modelling qty by OLS
       The estimation sample is: 2 - 111

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
qty_1                0.203549    0.09406     2.16  0.0327   0.0416
Constant              6.78385     0.8049     8.43  0.0000   0.3968

sigma                0.731432  RSS                 57.7792493
R^2                 0.0415598  F(1,108) =     4.683 [0.033]*
Adj.R^2             0.0326853  log-likelihood        -120.671
no. of observations       110  no. of parameters            2
mean(qty)             8.51915  se(qty)               0.743687


AR 1-2 test:      F(2,106)  =   1.9872 [0.1422]
ARCH 1-1 test:    F(1,108)  =   2.0874 [0.1514]
Normality test:   Chi^2(2)  =   6.9103 [0.0316]*
Hetero test:      F(2,107)  =   3.6890 [0.0282]*
Hetero-X test:    F(2,107)  =   3.6890 [0.0282]*
RESET23 test:     F(2,106)  =  0.69995 [0.4989]
```

Interested in **effects of weather** on quantity sold:

- Estimate auto-regressive model with weather variables added in
- Include: Stormy, Rainy, Cold
- Which variables are individually significant?
- Which variables are jointly significant?
- Do weather variables explain outliers in previous model?

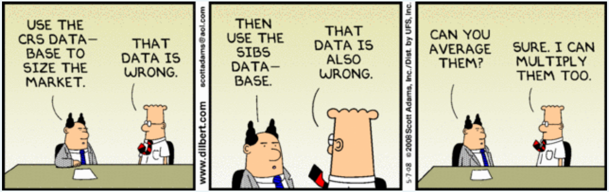
EQ(17) Modelling qty by OLS

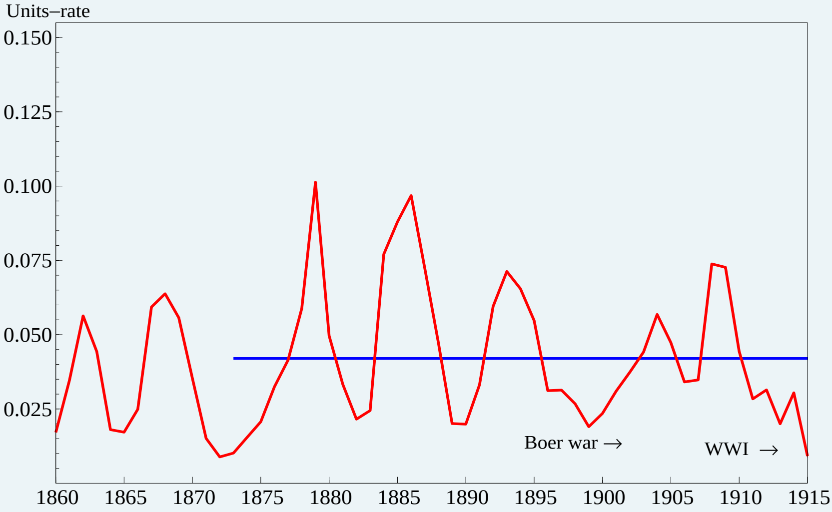
	Coefficient	Std.Error	t-value	t-prob	Part.R^2
qty_1	0.184254	0.09336	1.97	0.0511	0.0358
Constant	7.06086	0.8097	8.72	0.0000	0.4200
stormy	-0.342175	0.1681	-2.04	0.0443	0.0380
rainy	0.0824118	0.1918	0.430	0.6683	0.0018
cold	-0.0566163	0.1524	-0.372	0.7109	0.0013

sigma	0.721793	RSS	54.7034867
R^2	0.0925804	F(4,105) =	2.678 [0.036]*
Adj.R^2	0.0580121	log-likelihood	-117.663
no. of observations	110	no. of parameters	5
mean(qty)	8.51915	se(qty)	0.743687

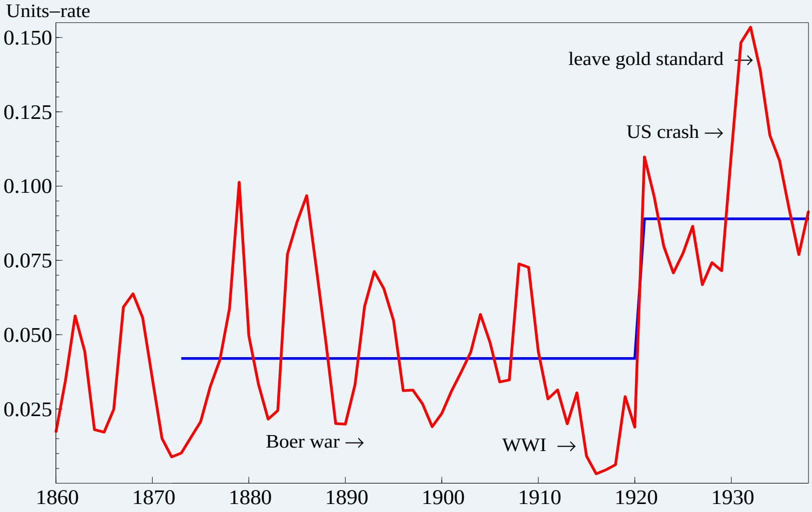
AR 1-2 test:	F(2,103)	=	0.82520 [0.4410]
ARCH 1-1 test:	F(1,108)	=	1.7838 [0.1845]
Normality test:	Chi^2(2)	=	8.7179 [0.0128]*
Hetero test:	F(5,104)	=	1.3869 [0.2352]
Hetero-X test:	F(5,104)	=	1.3869 [0.2352]
RESET23 test:	F(2,103)	=	0.65843 [0.5198]

- Load data:
  UKHist2015_metrics.in7/`UKHist2015_metrics.bn7`
- Graph UK unemployment rate 'Ur'

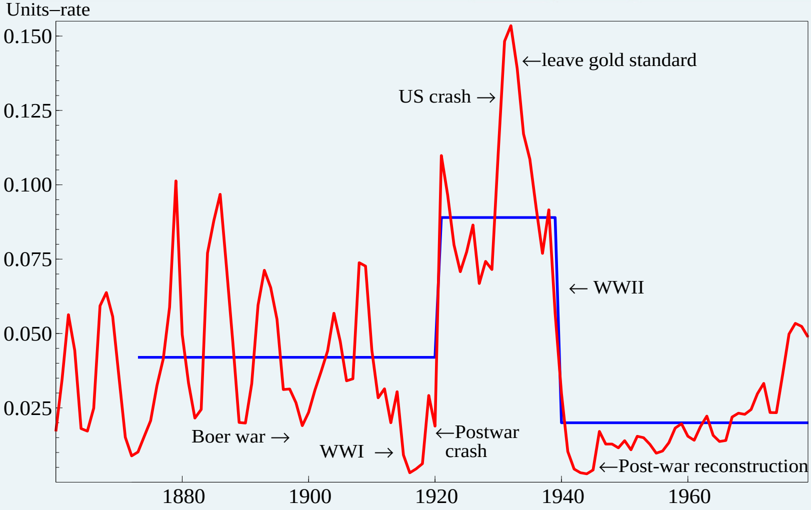




Clear business cycle before World War I.

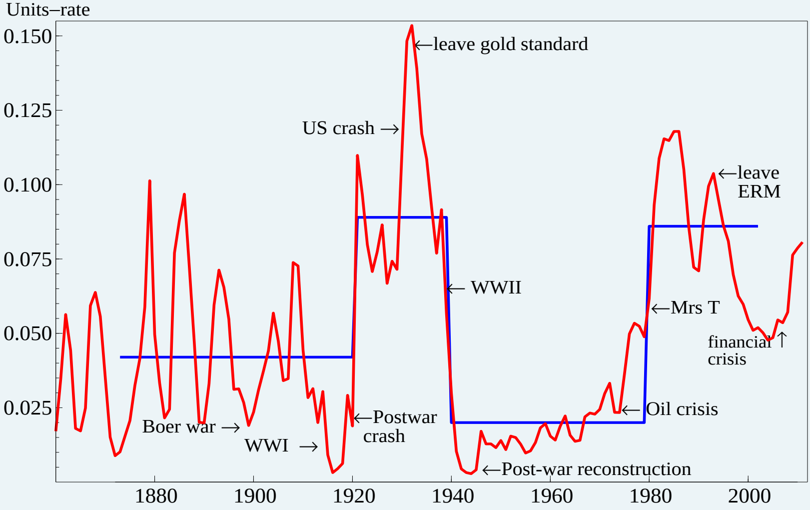


Leaps after WWI.



Rapid drop at WWII, then steady through the post-war reconstruction, but:

Events affecting UK unemployment 1873–2011



Wrecked by the oil crisis and Mrs Thatcher–then financial crisis: unlike inflation, shows only 4 distinct epochs.

Do not have complete and correct economic theories from which to derive 'correct' statistical models. As do not know DGP, must postulate theory-based statistical model.

Two hypothetical models of UK unemployment rate $U_{r,t}$:

- **First** is that a high wage share causes unemployment as labour 'too expensive'.
- **Second** is that high unemployment leads to high unemployment from 'discouraged workers'.

Formulate first as the linear regression:

$$U_{r,t} = \beta_0 + \beta_1 (w_t - p_t - g_t + l_t) + \epsilon_t \tag{9}$$

and the second becomes the autoregression:

$$U_{r,t} = \gamma_0 + \gamma_1 U_{r,t-1} + \nu_t \tag{10}$$

Both are 'straw' examples to illustrate how **not** to proceed.

Estimate both models:

1. Static theory model (Wage Share):

$$U_{r,t} = \beta_0 + \beta_1(w_t - p_t - g_t + l_t) + \epsilon_t \qquad (11)$$

2. Autoregression:

$$U_{r,t} = \gamma_0 + \gamma_1 U_{r,t-1} + \nu_t \qquad (12)$$

- Store and plot the residuals
- Comment on the results.

Estimation of static model (11) yields:

$$\widehat{U}_{r,t} = \underset{(0.06)}{-0.14} - \underset{(0.06)}{0.19} \left(w_t - p_t - g_t + l_t\right)$$

$$R^2 = 0.075 \;\; \widehat{\sigma}_\epsilon = 0.033 \;\; T = 1860 - 2011 \qquad (13)$$

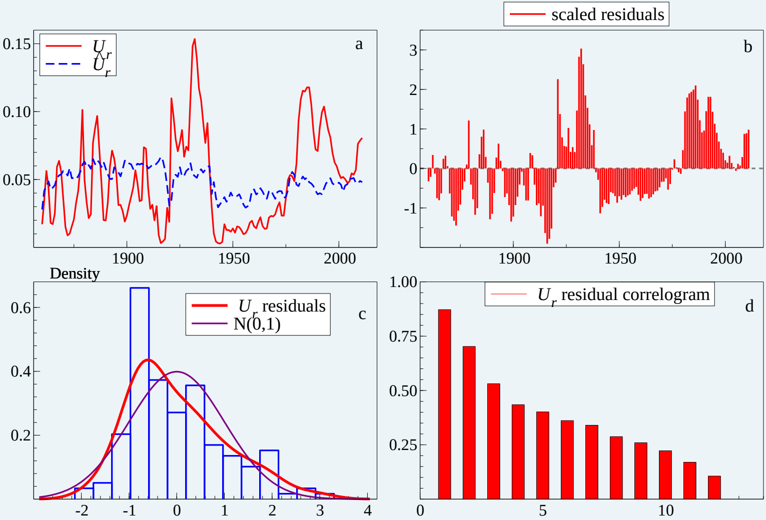
Estimates ‘seem significant’–in that the $t_{\beta_i=0}$ statistics reject their null hypotheses–but will question that shortly.

If so, a high wage share **lowers** unemployment,
which is the ‘wrong’ sign.

The fit is very poor: $R^2 = 0.078$ suggests most of movements in unemployment are not explained by the model.

...numerous problems shown in next Figure.

Wage share model of UK unemployment



- Panel a shows the movements in the fitted line, $\widehat{U}_{r,t}$, namely $0.20(w_t - p_t - g_t + l_t)$, which does not have the correct 'time series profile' to explain unemployment.
- The scaled residuals, $(U_{r,t} - \widehat{U}_{r,t})/\widehat{\sigma}_{\epsilon}$, in panel b move systematically and are far from 'random.

  Panel d shows their correlogram:
  highly positively autocorrelated as far back as 10 years.
- Panel c plots the residual histogram, with an estimate of the density and a normal density for comparison.
  There is 'ocular' evidence of some non-normality.

Now consider the performance of the 'rival' auto-regressive model.

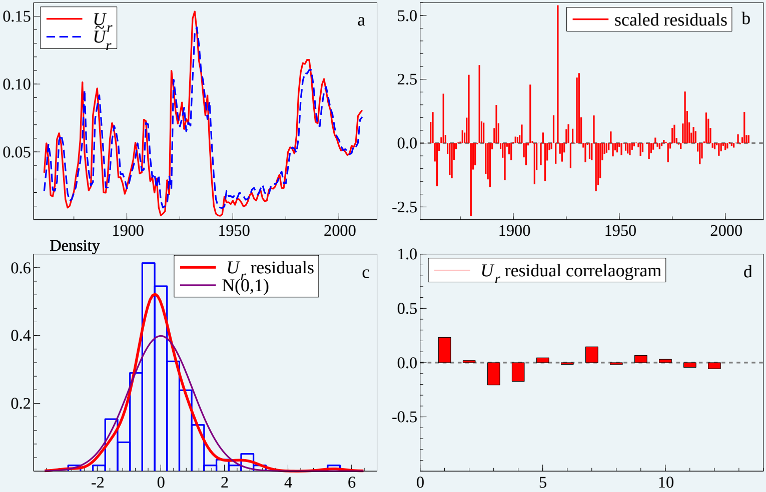
Estimation of Autoregression yields :

$$\widetilde{U}_{r,t} = \underset{(0.002)}{0.006} + \underset{(0.04)}{0.88} \, U_{r,t-1}$$

$$R^2 = 0.78 \quad \widehat{\sigma}_v = 0.016 \qquad (14)$$

- The fit is much better, $R^2 = 0.78$: some movements in unemployment are explained by (14)–next Figure panel a.
- The residuals in panel b are less systematic, but there is a large ‘spike’ or ‘outlier’ in 1920, **even though least squares tries to minimize squared residuals**, so there is nothing in the model to explain that jump in unemployment.
- The residual correlogram in panel d is much ‘flatter’ than for (13), and the residual histogram in panel c is closer to the normal density, with a large outlier.

Autoregressive model of UK unemployment



So far:

- Static theory: mis-specified, misses dynamics, low-explanatory power.
- Autoregressive model: no real insight aside from persistence.
    - E.g. how to construct counter-factuals?

Now: build improved model of UK unemployment.

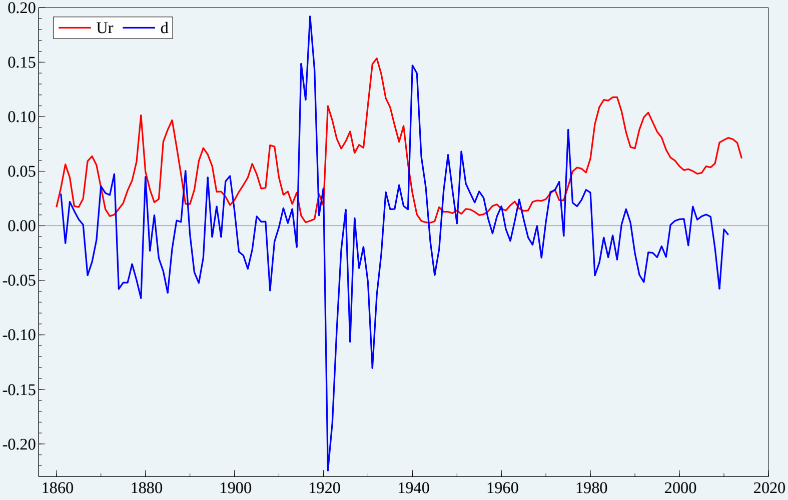
- **Do:** Could include wage share $(w - p - g + l)_t$ and its lagged value in the autoregressive model of $U_{r,t}$.
  $\rightarrow$ Adds little: $R^2 = 0.79$ when it was 0.78.
- Instead, will assume (and test) employment increases when hiring is profitable, and falls if not.
- No good data on profit changes, so use a 'proxy'–namely a variable that is usually closely related.

Proxy variable:

- Changes in revenues are linked to changes in GDP: Δg_t.
- Capital costs depend on real borrowing costs: $(R_L - \Delta p)_t$.

Approximate changes in profits by the difference between the proxies for costs and for revenues: $d_t = [\Delta g_t - (R_L - \Delta p)_t]$.

Profits proxy and unemployment



Build a model of the unemployment rate using:

- Lagged unemployment
- The Profit-proxy and its lag

Interpret:

- Store and plot the residuals
- Comment on the results.
- How can this model be interpreted?

The paths of the two time series have much in common: so let's model $U_{r,t}$ using d_t:

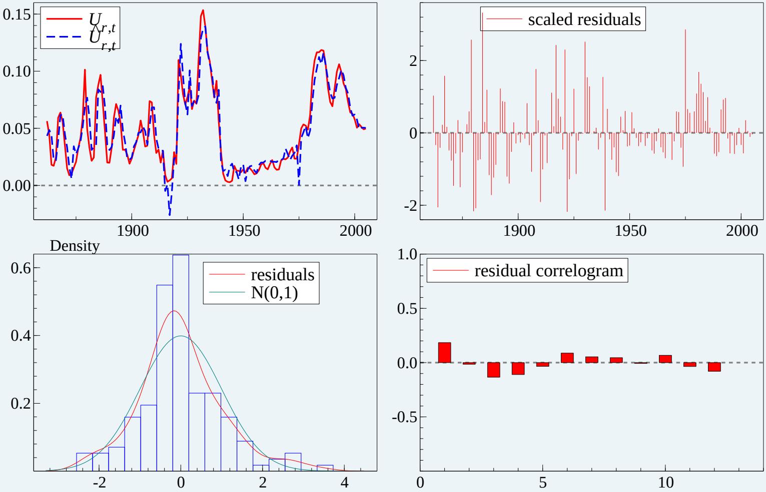
$$\widehat{U}_{r,t} = \underset{(0.002)}{0.007} + \underset{(0.035)}{0.86} U_{r,t-1} - \underset{(0.024)}{0.243} d_t + \underset{(0.023)}{0.095} d_{t-1}$$

$$R^2 = 0.86 \quad \widehat{\sigma}_\epsilon = 0.013 \tag{15}$$

The fit is better than either previous model, and the impacts of both d_t and its lag are statistically significant:

Next Figure records the actual $U_{r,t}$ and fitted $\widehat{U}_{r,t}$ values, residuals $\widehat{\epsilon}_t = U_{r,t} - \widehat{U}_{r,t}$, their density and correlogram.

Graphics of dynamic unemployment model



We have seen static equations of the form:

$$y_t = \beta_0 + \beta_1 z_t + \epsilon_t \tag{16}$$

and autoregressive equations such as:

$$y_t = \gamma_0 + \gamma_1 y_{t-1} + \nu_t \tag{17}$$

so combine these in a more general dynamic model:

$$y_t = \beta_0 + \beta_1 z_t + \beta_2 y_{t-1} + \beta_3 z_{t-1} + \epsilon_t \tag{18}$$

In (18), y_t responds to changes in z_t, in its own lag, or previous value, y_{t-1}, and to the lag z_{t-1}, that relation being perturbed by a random error $\epsilon_t \sim \mathsf{IN}[0, \sigma_\epsilon^2]$.

Thus final model adds inter-dependence (z_t) to dynamics (y_{t-1}, z_{t-1}).

To interpret our model, transform it to **Equilibrium-Correction Form**:

- Subtracting y_{t-1} from both sides to create the first difference on the left-hand side:

$$y_t - y_{t-1} = \beta_0 + \beta_1 z_t + (\beta_2 - 1)\, y_{t-1} + \beta_3 z_{t-1} + \epsilon_t \quad (19)$$

- Next, subtract $\beta_1 z_{t-1}$ from $\beta_1 z_t$, to create a difference, and add it to $\beta_3 z_{t-1}$ (to keep the equation balanced):

$$\Delta y_t = \beta_0 + \beta_1 \Delta z_t - (1 - \beta_2)\, y_{t-1} + (\beta_1 + \beta_3)\, z_{t-1} + \epsilon_t \quad (20)$$

which reveals that β_1 is the impact of Δz_t on Δy_t.

- Now collect the terms in y_{t-1} and z_{t-1} when $|\beta_2| < 1$ as:

$$\Delta y_t = \beta_0 + \beta_1 \Delta z_t - (1 - \beta_2)\,(y_{t-1} - \kappa_1 z_{t-1}) + \epsilon_t \quad (21)$$

where $\kappa_1 = (\beta_1 + \beta_3)/(1 - \beta_2)$.

Convenient to collect the intercept with the last term as well:

$$\Delta y_t = \beta_1 \Delta z_t - (1-\beta_2)(y_{t-1} - \kappa_0 - \kappa_1 z_{t-1}) + \epsilon_t \qquad (22)$$

where $\kappa_0 = \beta_0/(1-\beta_2)$.

Interpretation:

When change ceases, so $\Delta y_t = \Delta z_t = 0$, or $y_t = y_{t-1} = y$ and $z_t = z_{t-1} = z$, with no shocks, so $\epsilon_t = 0$, then $y = \kappa_0 + \kappa_1 z$, which is the **equilibrium**.

The model in is called an **'equilibrium-correction' mechanism** (often abbreviated to **EqCM**) as the change in y_t 'corrects' to the previous deviation $(y_{t-1} - \kappa_0 - \kappa_1 z_{t-1})$ from equilibrium at a rate depending on $(1-\beta_2)$.

Do: PcGive can solve for EqCMs by `test, dynamic analysis, static long-run solution and lag structure analysis`.

For unemployment model, find EqCM$= U_r - 0.049 + 1.06d$

Construct new variable EqCM = $U_r - 0.049 + 1.06d$

Checking: $\beta_0 = 0.007$, $\beta_2 = 0.86$, $\beta_1 = -0.243$, and $\beta_3 = 0.095$

- $\kappa_1 = (\beta_1 + \beta_3)/(1 - \beta_2) = (-0.243 + 0.095)/0.14 = -1.06$
- $\kappa_0 = \beta_0/(1 - \beta_2) = 0.007/0.14 = 0.05$

Thus, rounding the two coefficients, the equilibrium in (15) is:

$$U_r = 0.05 - d$$

Rounding the two coefficients, the equilibrium in (15) is:

$$U_r = 0.05 - d$$

or **5% unemployment** when $d = 0$ (which is its mean).

Do: We can reformulate the equation as:

$$\Delta\widehat{U}_{r,t} = -0.24\Delta d_t - 0.14\left(U_{r,t-1} - 0.05 + d_{t-1}\right) \quad (23)$$

- Unemployment falls or rises by approximately 1% for every 1% increase or decrease in d = $[\Delta g - (R_L - \Delta p)]$.
- Immediate effect of a change in d is an impact of $\pm 0.24\%$, so unemployment only moves part of the way to the eventual impact of 1% and that creates a disequilibrium.
- Then 14% of that deviation from equilibrium is removed each period.

Although our dynamic model is sensible and interpretable, it has an important restriction:

We only allowed for 1 lag, so excluded lagged changes like $\Delta U_{r,t-1}$ and Δd_{t-1} (or longer).

Do:

- Add $\Delta U_{r,t-1}$ and Δd_{t-1} to our equilibrium correction model.

Although our dynamic model is sensible and interpretable, it has an important restriction:

We only allowed for 1 lag, so excluded lagged changes like $\Delta U_{r,t-1}$ and Δd_{t-1} (or longer).

Those are easily added, and doing so delivers:

$$\Delta \widehat{U}_{r,t} = \underset{(0.07)}{0.17} \Delta U_{r,t-1} - \underset{(0.02)}{0.24} \Delta d_t - \underset{(0.03)}{0.12} \left(U_{r,t-1} - 0.05 + d_{t-1} \right)$$

$$(R^*)^2 = 0.47 \quad \widehat{\sigma}_\epsilon = 0.012 \tag{24}$$

$\widehat{\sigma}_\epsilon$ is smaller, so the model is an improvement.
Adding $\Delta U_{r,t-1}$ was significant, but Δd_{t-1} was not.

- When the real long-term interest rate, $R_L - \Delta p$, equals the real growth rate, Δg, so $d = 0$, equilibrium unemployment is about 5%, close to the average unemployment rate.
- The model does not explain why, merely that movements of U_r away from that rate are associated with non-zero values of d.
- To lower unemployment and return towards that equilibrium requires lower real long-term interest rates or faster growth (higher d): both are policies currently in force, but difficult to maintain.

Modelling log consumption (CONS) as a function of log income (INC) using "cons.in7":

- Estimate an AR(1) model of consumption. What is the long-run equilibrium?
- Estimate the following model:

$$CONS_t = \alpha_0 + \alpha_1 CONS_{t-1} + \beta_1 INC_t + \beta_2 INC_{t-1} + u_t$$

- Re-parametrise the model and express it in equilibrium correction form:

$$\Delta CONS_t = \beta_1 \Delta INC_t + \gamma \left(CONS_{t-1} - \lambda INC_{t-1} - \phi \right) + u_t$$

- How do the coefficients γ, λ, ϕ relate to the original coefficients α_0, α_1, β_0, β_1, β_2?
- What is the immediate effect of an increase in income on consumption?
- What is the long-run equilibrium relationship between consumption and income in your estimated model?
- How quickly does consumption respond to changes in income?
- Based on the diagnostic tests, is your model well-specified?
- Estimate a more general model including multiple lags, seasonal dummy variables, and a linear trend. How could you go about simplifying the model and reducing the number of variables?