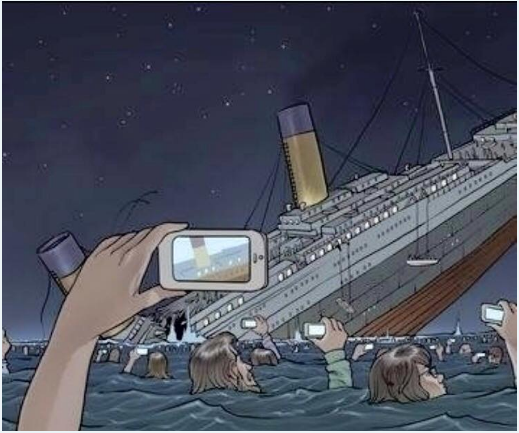
Would you have survived the sinking of the Titanic?



Econometrics: Computer Modelling

Felix Pretis

Programme for Economic Modelling
Oxford Martin School, University of Oxford

Lecture 2: Micro-Econometrics: Limited Dep. Variable Models

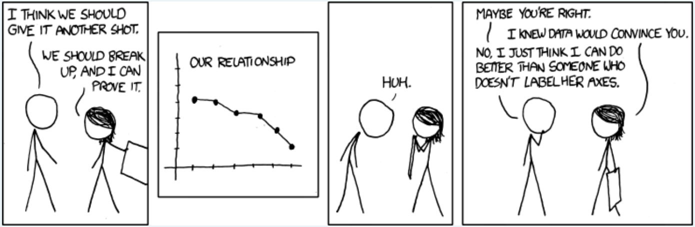
- 1: Intro to Econometric Software & Cross-Section Regression
- 2: **Micro-Econometrics: Limited Indep. Variable**
- 3: Macro-Econometrics: Time Series

Last time:

- Introduce econometric modelling in practice
- Introduce OxMetrics/PcGive Software

Today:

- Binary dependent variables & Count data
  - Probability of being accepted into a Masters/PhD Programme (between [0,1])
  - Number of arrests (count)
  - Prob. of surviving the Titanic sinking, participating in labour force
- Additional functions in OxMetrics/PcGive



- Economies high dimensional, interdependent, heterogeneous, and evolving: comprehensive specification of all events is impossible.
- Economic Theory
  - likely wrong and incomplete
  - **meaningless** without empirical support

  - Econometrics to discover new relationships from data
  - Econometrics can provide empirical support. . . or refutation.

Structure of data

	admit	gre	gpa	rank	rank1	rank2	rank3	rank4
1	0	380	3.61	3	0	0	1	0
2	1	660	3.67	3	0	0	1	0
3	1	800	4	1	1	0	0	0
4	1	640	3.19	4	0	0	0	1
5	0	520	2.93	4	0	0	0	1
6	1	760	3	2	0	1	0	0
7	1	560	2.98	1	1	0	0	0
8	0	400	3.08	2	0	1	0	0
9	1	540	3.39	3	0	0	1	0
10	0	700	3.92	2	0	1	0	0
11	0	800	4	4	0	0	0	1
12	0	440	3.22	1	1	0	0	0
13	1	760	4	1	1	0	0	0

Data on **admission to graduate school** (US) as a function of:

- GPA
- GRE score
- Rank of undergraduate institution

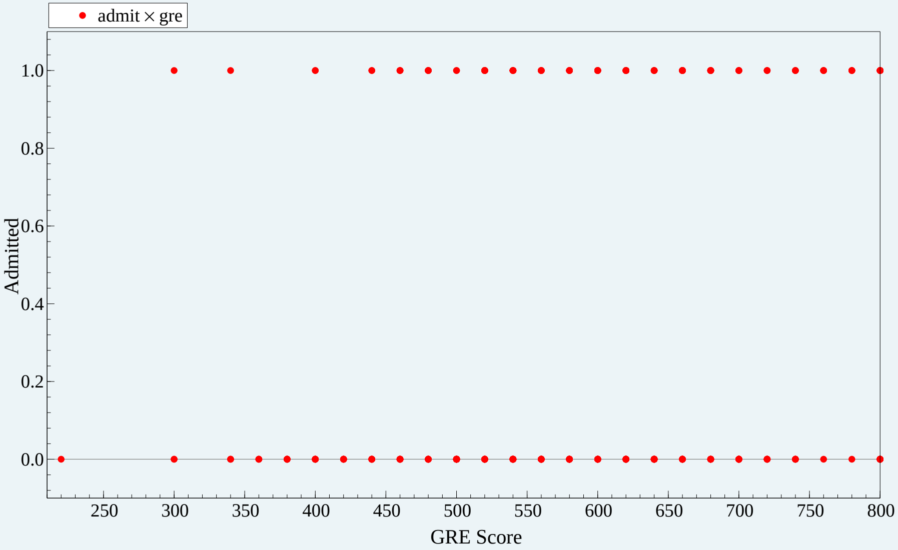
Dataset: ”gradschool.xlsx”

Other file formats? Datasets: .in7 & .bn7 files

Build a **Linear Probability Model** (LPM) for gradschool admission:

- Create a new database in OxMetrics
    - Go to File, New, OxMetrics Data
    - Set start period = 1 (undated for cross-sectional data)
    - Copy & Paste Data from Excel file: ”gradschool.xlsx”
    - Save as .in7 data file on your computer
- Or open .csv in OxMetrics
- Construct appropriate variables to take the rank of the university into account
    - *Algebra*: `rank1 = (rank == 1) ? 1 : 0;`
      creates a dummy variable = 1 if rank==1
- Plot the observed and predicted values against GRE. Based on the model output highlight shortcomings of the LPM

Plot the data!



Fit a simple Linear probability model (OLS)

```
EQ( 1) Modelling admit by OLS-CS
       The dataset is: gradschool.in7
       The estimation sample is: 1 - 400

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant            -0.258910     0.2160    -1.20  0.2314   0.0036
gre               0.000429572  0.0002107     2.04  0.0422   0.0104
gpa                  0.155535    0.06396     2.43  0.0155   0.0148
rank2               -0.162365    0.06771    -2.40  0.0170   0.0144
rank3               -0.290570    0.07025    -4.14  0.0000   0.0416
rank4               -0.323026    0.07932    -4.07  0.0001   0.0404

sigma                0.444866  RSS                 77.9750245
R^2                  0.100401  F(5,394) =     8.795 [0.000]**
Adj.R^2             0.0889844  log-likelihood          -240.56
no. of observations       400  no. of parameters             6
mean(admit)            0.3175  se(admit)              0.466087

Normality test:   Chi^2(2)  =   212.46 [0.0000]**
Hetero test:      F(7,392)  =   3.8513 [0.0005]**
Hetero-X test:    F(8,391)  =   3.6183 [0.0004]**
RESET23 test:     F(2,392)  =  0.19773 [0.8207]
```

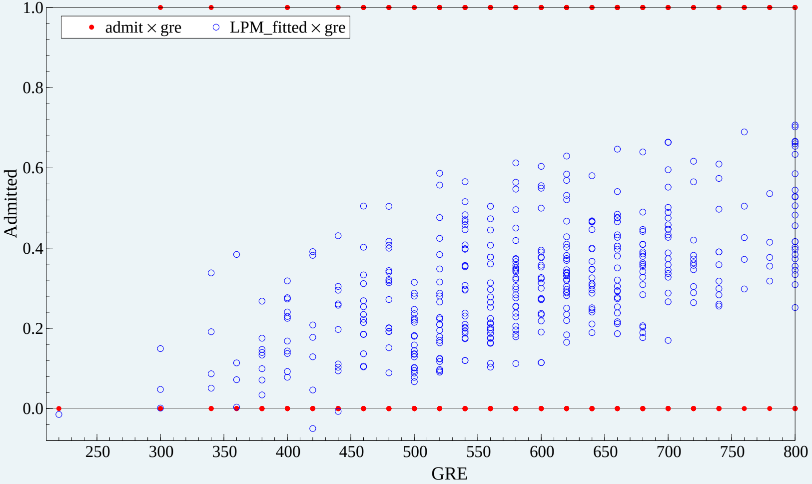
Concerns with Linear Probability Model

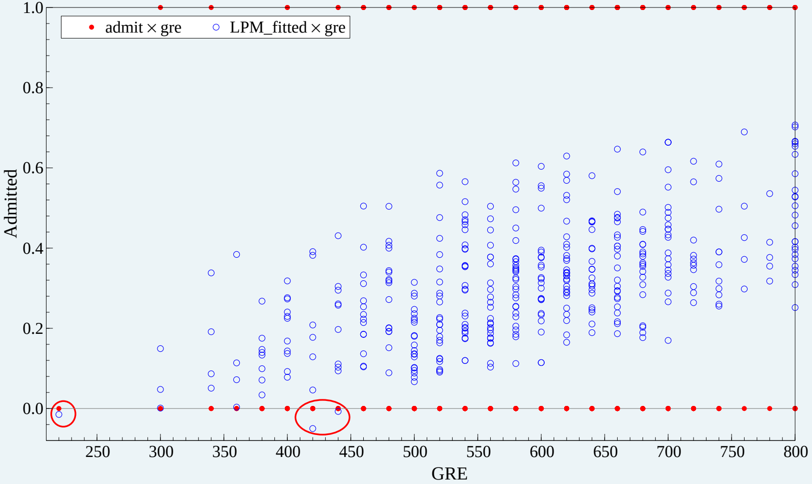
- Assumes continuous dep. variable & constant effect of covariates on probability of success (could exceed 1)
- Predicted values outside [0,1] range: *Test* - *Store...*
- Heteroskedasticity by construction:

$$P(y=1|x) = x'\beta + u \quad (1)$$

$$V(u|x) = x'\beta(1-x'\beta) \quad (2)$$

Fitted Values of LPM




admit × gre
LPM_fitted × gre
Admitted
GRE
1.0
0.8
0.6
0.4
0.2
0.0
250
300
350
400
450
500
550
600
650
700
750
800

Binary response variable, link function $G(\cdot)$

$$P(y=1|x) = G(\beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k) = G(\beta_0 + \mathbf{x}\beta) \tag{3}$$

- **Probit**:

$$P(y=1|x) = \Phi(x'\beta) \tag{4}$$

  $\Phi(\cdot)$ is the standard normal distribution function.

- **Logistic Regression**:

$$P(y=1|x) = \frac{\exp(\beta_0 + \mathbf{x}\beta)}{1 + \exp(\beta_0 + \mathbf{x}\beta)} \tag{5}$$

- Maximum Likelihood Estimation
- No analytical solution

1) Log-Odds Ratio

Note that the odds ratio (probability of success over probability of failure) in the logit model is given as:

$$\frac{P(y=1|x)}{1-P(y=1|x)} = \exp(\beta_0 + \mathbf{x}\beta) \tag{6}$$

Therefore, taking logs:

$$\log\left(\frac{P(y=1|x)}{1-P(y=1|x)}\right) = \beta_0 + \mathbf{x}\beta \tag{7}$$

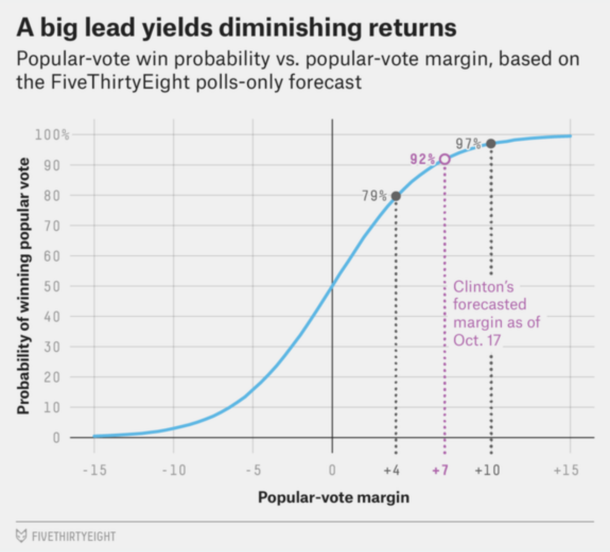
Thus, $100 \times \beta_k$ has the interpretation as % increase in odds ratio for a one-unit increase in x_k

2) Marginal Effects (ME)

$$\frac{\partial P(y=1|x)}{\partial x_k} = \frac{\partial}{\partial x_k}\left(\frac{\exp(\beta_0 + \mathbf{x}\beta)}{1+\exp(\beta_0+\mathbf{x}\beta)}\right) \tag{8}$$

$$= \beta_k P(y=1|x)\left(1 - P(y=1|x)\right) \tag{9}$$

- ME_k same sign as coefficient β_k
- Marginal effects are largest when $P = 0.5$, i.e. largest for individuals whose outcomes have the highest variance, $p(1-p)$.



goo.gl/LUD7ft

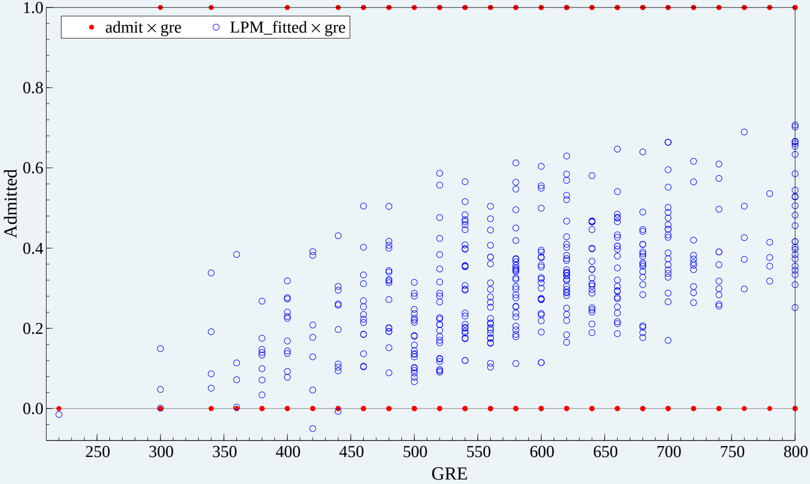
- *Models for Discrete Data*
- *Binary Discrete Choice using PcGive*
- *Logit*
- *Newton's Method* (no analytical solution – numerical algorithm)

What are the effects of rank, gpa, gre, on the probability of being admitted to Grad School?

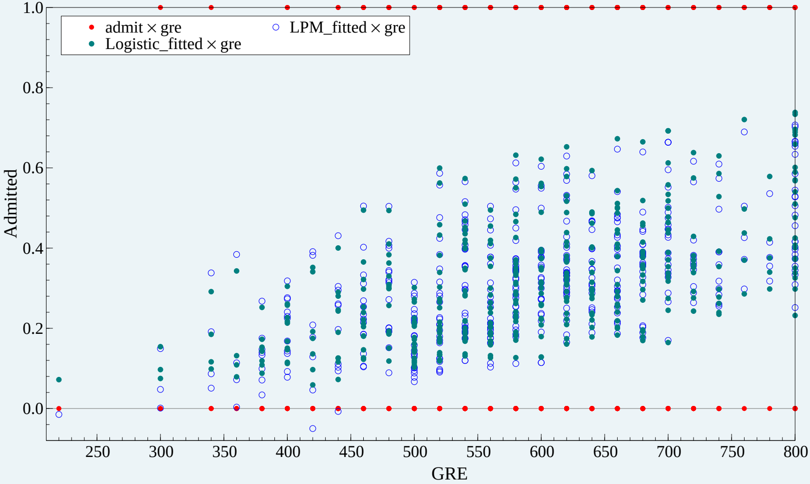
Logistic Model

```
CS( 1) Modelling admit by LOGIT
       The dataset is: gradschool.in7
       The estimation sample is 1 - 400

                  Coefficient  Std.Error  t-value  t-prob
Constant             -3.98998      1.140    -3.50   0.001
gre                0.00226443   0.001094     2.07   0.039
gpa                  0.804038     0.3318     2.42   0.016
rank2               -0.675443     0.3165    -2.13   0.033
rank3                -1.34020     0.3453    -3.88   0.000
rank4                -1.55146     0.4178    -3.71   0.000

log-likelihood    -229.258746  no. of states                2
no. of observations       400  no. of parameters            6
baseline log-lik    -249.9883  Test: Chi^2(  5)        41.459 [0.0000]**
AIC                470.517492  AIC/n               1.17629373
mean(admit)            0.3175  var(admit)            0.216694
Newton estimation (eps1=0.0001; eps2=0.005): Strong convergence

              Count   Frequency Probability    loglik
State 0         273     0.68250     0.68250    -97.40
State 1         127     0.31750     0.31750    -131.9
Total           400     1.00000     1.00000    -229.3
```

Fitted Values of LPM



Fitted Values of Logistic



Replicability is important!

Easy to make mistakes/forget what you have done. Code to reproduce your modelling:

- Batch code (intuitive code, similar to STATA do-files)
- Ox code (matrix programming language, similar to Matlab, R)

Batch Code:

- .fl files
- ALT+B: Batch code for last model

```
//Lecture 2: Micro-Econometrics, Limited Dependent Variable

//Linear Probability Model

module("PcGive");
package("PcGive", "Cross-section");
usedata("gradschool.in7");
system
{
    Y = admit;
    Z = Constant, gre, gpa, rank2, rank3, rank4;
}
estimate("OLS-CS", 1, 1, 400, 1);

//Logistic Regression Model

module("PcGive");
package("LogitJD", "Binary");
usedata("gradschool.in7");
system
{
    Y = admit;
    X = gre, gpa, rank2, rank3, rank4;
    F = Constant;
}
estimate("LOGIT", 1, 1, 400, 1);
```

Using **Batch Code**, estimate and store the following models for Gradschool admissions:

1. A linear probability model without an intercept with a different base rank
2. A logistic regression without GPA variable and using observations for the individuals $i = 50, ..., 200$.
3. In the form of comments in batch code, add the results of a test that all rank variables can be dropped from the model.

Estimate:

- LPM of *admit* on *constant* and *rank1*
- Logit Model of *admit* on *constant* and *rank1*

Compare predicted values between the two models.

- **LPM** of *admit*:

The estimation sample is: 1 - 400

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
Constant	0.277286	0.02481	11.2	0.0000	0.2388
rank1	0.263697	0.06354	4.15	0.0000	0.0415

Predicted: $0.277 + 0.26\mathrm{I}_{\{\text{Rank=1}\}}$

= 0.54 (for Rank = 1)

- **Logit** of *admit*:

	Coefficient	Std.Error	t-value	t-prob
Constant	-0.957963	0.1213	-7.90	0.000
rank1	1.12227	0.2841	3.95	0.000

Predicted: $\frac{exp\left(-0.957+1.12\mathrm{I}_{\{\text{Rank=1}\}}\right)}{\left(1-exp\left(-0.957+1.12\mathrm{I}_{\{\text{Rank=1}\}}\right)\right)}$

= 0.277 (for Rank $\neq$ 1) and = 0.54 (for Rank = 1)

So far: Binary dependent variable [0,1]
Now: Count data – Poisson regression

Poisson Regression

- Dependent Variable: non-negative integers 0,1,2...
- $y \sim \text{Poisson}(\mu)$
- linear model not ideal (as before)

Model expected value as exponential function:

$$y_i = E[y_i|x_i] + u_i \tag{10}$$

$$E[y_i|x] = \exp(\beta_0 + \beta_1 x_{1,i} + \cdots + \beta_k x_{k,i}) \tag{11}$$

$$y_i = e^{x_i'\beta} + u_i \tag{12}$$

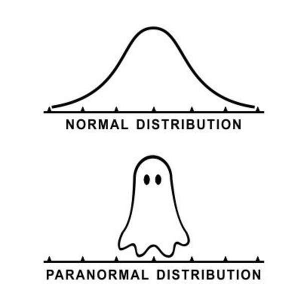
Interpretation:

- Approx: $100\beta_k \Delta x_k \approx \%\Delta E[y_i|x]$
- Exact proportional change: $\exp(\beta_k \Delta x_k) - 1$

Count data: 0, 1, 2, . . . , modelled as **Poisson Distribution** with λ_i:

$$
\begin{aligned}
E[y_i|x] &= \lambda_i = \exp(\beta_0 + \beta_1 x_{1,i} + \cdots + \beta_k x_{k,i}) \\
V[y_i|x] &= E[y_i|x] \\
P\left(Y = y_i|\lambda_i(x_i)\right) &= \frac{e^{-\lambda_i}\lambda_i^{y_i}}{y_i!}
\end{aligned}
$$

Estimation using Maximum Likelihood.



Modelling Number of Arrests:

- Number of times a man is arrested in 1986: ‘narr86’
- ”arrests.in7”
- Plot the data!

Poisson Regression:

- *Models for Discrete Data*
- *Count Data using PcGive*

Model:

- Dependent variable: ”narr86”
- Independent variables:
    - ”pcnv” (prop. of prior arrests that led to conviction)
    - ”avgsen” (avg sentence length)
    - ”tottime” (time in prison since 18)
    - ”ptime86” (months spent in prison)
    - ”qemp86” (quarters employed)
    - ”inc86” (income)
    - ”black”, ”hispan”

OXFORD MARTIN SCHOOL UNIVERSITY OF OXFORD Institute for New Economic Thinking AT THE OXFORD MARTIN SCHOOL

```
CS( 1) Modelling narr86 by POISSON
       The dataset is: arrests.in7
       The estimation sample is 1 - 2725

                    Coefficient  Std.Error  t-value  t-prob
Constant              -0.617178    0.06365    -9.70   0.000
pcnv                  -0.405258    0.08488    -4.77   0.000
avgsen               -0.0236365    0.01993    -1.19   0.236
tottime               0.0243425    0.01476     1.65   0.099
ptime86              -0.0985944    0.02071    -4.76   0.000
qemp86               -0.0361131    0.02892    -1.25   0.212
inc86               -0.00814627   0.001038    -7.85   0.000
black                  0.660356    0.07383     8.94   0.000
hispan                 0.499594    0.07392     6.76   0.000

log-likelihood     -2249.08013  not truncated
no. of observations       2725  no. of parameters          9
baseline log-lik     -2441.921  Test: Chi^2(  8)      385.68  [0.0000]**
AIC                 4516.16026  AIC/n             1.65730652
mean(narr86)          0.404404  var(narr86)         0.737742
```

- Store the batch code as ".fl" file.

1. What is the effect of being black/hispanic on the number of arrests?
2. Manually conduct a likelihood ratio test of: excluding *black*, *hispan*
    - Run Models in batch file.
    - $LR = -2\left[\ln\left(\hat{L}_R\right) - \ln\left(\hat{L}_{UR}\right)\right] \sim \chi^2_q$
    - χ^2_2: 5% Critical value is 5.99

Surviving the Titanic

What is your estimated probability of survival?

- Create variables that measure the cabin class (& clean data)
- Create a new database using ”titanic_data.csv”
- Estimate the probability of survival (”survived”) using
  - Cabin class
  - ”sex”: =1 if female
  - ”age”: in years
  - ”num_sibs_sp”: number of siblings or spouses on board
  - ”num_par_ch”: number of parents or children on board

Answering the following questions:

1. What is the unconditional probability of survival?
2. What is the average survival rate for each class?
3. Estimate the following models using **three** alternative methods and compare the results
   - Create batch file for your models & plots illustrating your results.
   - What is the effect of cabin class/sex/age/having siblings or kids on-board on the probability of survival? How can the coefficients be interpreted? What difference do you find between the two methods used?
   - What is your personal probability of survival for your assigned cabin class, given that you assume your parents were not on board, but your siblings/spouses (if you have any) would have been?
4. What determines the number of siblings people had on board?
   - Construct a test for class not affecting the number of siblings.

OxMetrics and PcGive Exercise: Female labour force participation.

- Create a new database using ”labourforce.xlsx”
- ”inlf” binary variable =1 if married woman in labour force in 1975.

hours	hours worked, 1975
kidslt6	# kids < 6 years
kidsge6	# kids 6–18
age	woman’s age in yrs
educ	years of schooling
wage	estimated wage from earns., hours
repwage	reported wage at interview in 1976
hushrs	hours worked by husband, 1975
husage	husband’s age
huseduc	husband’s years of schooling
huswage	husband’s hourly wage, 1975
faminc	family income, 1975
mtr	fed. marginal tax rate facing woman
motheduc	mother’s years of schooling
fatheduc	father’s years of schooling
unem	unem. rate in county of resid.
city	=1 if live in city
exper	actual labor mkt exper

Answering the following questions:

1. Estimate models using two alternative methods and compare the results (create a batch file).
2. What is the effect of age/educ/experience/having kids on the probability of being in the labour force? What difference do you find between the two methods used?
3. Allow for diminishing marginal returns to experience. What are your findings?
4. Build a more general model, including additional covariates. Which ones are significant? How could you reduce the number of variables?
5. Using batch code, sequentially eliminate variables based on their significance (conduct backwards-elimination). What other model selection methods could you use? Advantages/disadvantages?
6. Classification: Hold back 200 observations, predict the labour force participation for the hold-back sample. What proportion are correctly classified? Build a model that achieves the highest classification rate.