General-to-Specific Time Series Modelling

Felix Pretis

University of Oxford

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Lecture 3: Exogeneity

Core References for Lecture 3:

- Engle, Hendry, and Richard (1983)* – 'Exogeneity' (EHR)
- Ericsson, Hendry, and Mizon (1998)* – Exogeneity, Cointegration, and Policy Analysis
- Hendry and Santos (2010)* – Automatic Test for Super Exogeneity
- Engle and Hendry (1993) – Super Exogeneity
- Hendry (2017) – Granger (Non-)Causality

Exogeneity: valid conditioning.

- Weak Exogeneity: conditions to study conditional model alone (relative to parameters of interest)
- Strong Exogeneity: feedback and conditional projections/forecasts.
- Super Exogeneity: invariance, policy analysis.

Conditions of variables in/out of the model. Beyond pre-determinedness and ‘strict exogeneity’ which refer to properties of unobserved error term.

Let $x_t \in R^n$ denote a vector of observable random variables generated at time t with observations $(t = 1, \ldots, T)$.

$$X_t^1 = (x_1, \ldots, x_t)' \tag{1}$$

is a $t \times n$ matrix. Denote X_0 the matrix of initial conditions. Information at time t is:

$$X_{t-1} = \begin{bmatrix} X_0 \\ X_{t-1}^1 \end{bmatrix} \tag{2}$$

Process generating T observations is assumed continous and represented by the joint density function $D(X_T^1 | X_0, \theta)$ where θ in the interior of Θ is a vector of unknown parameters.

The vector x_t is partitioned into

$$x_t = \begin{pmatrix} y_t \\ z_t \end{pmatrix}, y_t \in R^p, z_t \in R^q, p + q = n \tag{3}$$

Factorize the joint density (as using $P(a, b) = P(a \mid b)P(b)$) as:

$$D(X_T^1|X_0, \theta) = \prod_{t=1}^{T} D(x_t|X_{t-1}, \theta) \tag{4}$$

Parameters of Interest ψ: may not be interested in all parameters θ. Instead interested in $\psi \to f(\theta)$. Consider therefore a one-to-one transformation:

$$h : \Theta \to \Lambda; \theta \to \lambda = h(\theta) \tag{5}$$

and partition λ into λ_1, λ_2.

Let Λ_i denote the set of admissible values of λ_i. Question is whether parameters of interest are functions of λ_i alone, whether there exists a function ϕ such that:

$$\phi : \Lambda_1 \to \Psi; \lambda_1 \to \psi = \phi(\lambda_1) \tag{6}$$

Partition the joint density into a **conditional** and **marginal** (no loss of information):

$$D(x_t|X_{t-1}, \lambda) = D(y_t|z_t, X_{t-1}, \lambda_1)D(z_t|X_{t-1}, \lambda_2) \qquad (7)$$

z_t is **weakly exogenous** (WE) over the sample period for parameters of interest ψ if and only if there exists a re-parametrisation with $\lambda = (\lambda_1, \lambda_2)$ such that:

- ψ is a function of λ_1 alone.
- λ_1 and λ_2 are variation free.

If z_t is WE, then can study conditional density alone to learn about ψ without having to consider marginal.

Likelihood framework (estimation) – Joint Likelihood (given initial conditions) denoted as:

$$L^0(\lambda; X_T^1) = L_1^0(\lambda_1; X_T^1) L_2^0(\lambda_2; X_T^1) \quad (8)$$

partitioned into conditional and marginal likelihood:

$$L_1^0(\lambda_1; X_T^1) = \prod_{t=1}^{T} D(y_t | z_t, X_{t-1}, \lambda_1) \quad (9)$$

$$L_2^0(\lambda_2; X_T^1) = \prod_{t=1}^{T} D(z_t | X_{t-1}, \lambda_2) \quad (10)$$

where under weak exogeneity estimates of parameter of interest ψ can be obtained by considering the conditional likelihood L_1^0 alone.

Granger Non-Causality:

Y^1_{t-1} does not Granger cause z_t with respect to X_{t-1} if and only if marginal density:

$$D(z_t|X_{t-1}, \theta) = D(z_t|Z_{t-1}, Y_0, \theta) \tag{11}$$

$\rightarrow$ past values of y_t do not enter the marginal density of z_t.

If this condition holds over the sample period, then the joint density $D(X^1_T|X_0, \theta)$ factorises as:

$$D(X^1_T|X_0, \theta) = \left[\prod_{t=1}^{T} D(y_t|z_t, X_{t-1}, \theta)\right] \left[\prod_{t=1}^{T} D(z_t|Z_{t-1}, Y_0, \theta)\right] \tag{12}$$

Strong Exogeneity:

z_t is strongly exogenous over the sample period for ψ if and only if it is weakly exogenous for ψ and:

- y does not Granger cause z

If z_t is strongly exogenous: we can conduct inference on parameters of interest in conditional model alone and create conditional forecasts on z_t (no feedback of y_t onto z_t).

Even if the parameters of the conditional density $D(y_t|z_t, X_{t-1}, \lambda_1)$ and marginal density $D(z_t|X_{t-1}, \lambda_2)$ are variation free, λ_1 may still change as λ_2 alters.

- Conditional model is structurally invariant if all its parameters λ_1 are invariant for any changes in the distribution of the conditioning variables. (Denote these changes as interventions C_2).
- E.g. $\lambda_{1,t} = \kappa\lambda_{2,t}$ with κ unknown, variation free but not invariant.

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Even if the parameters of the conditional density $D(y_t|z_t, X_{t-1}, \lambda_1)$ and marginal density $D(z_t|X_{t-1}, \lambda_2)$ are variation free, λ_1 may still change as λ_2 alters.

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z_t is **super exogenous** for ψ if z_t is weakly exogenous for ψ and the conditional model $D(y_t|z_t, X_{t-1}, \lambda_1)$ is structurally invariant.

Engle and Hendry (1983) on Super Exogeneity:

- Weak Exogeneity
- Parameter constancy: $\psi_t = \psi$ (or function of constant parameters)
- $\frac{\partial\lambda_1}{\partial\lambda_2} = 0 \quad \forall\lambda_2 \in C_2$

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Relating to "Endogeneity":

Commonly used definitions of exogeneity refer to properties of variables relative to an unobserved error term ϵ_t in regression.

Definition:

- z_t is predetermined if and only if: $z_t || \epsilon_{t+i} \quad \forall i \geqslant 0$
- z_t is strictly exogenous if and only if: $z_t || \epsilon_{t+i} \quad \forall i$

where $||$ denotes that processes are independent.

Neither predeterminedness nor strict exogeneity are sufficient (or necessary) to fulfil weak, strong, or super-exogeneity required for conditional modelling.

Exogeneity in Bivariate Normal:

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} \sim \mathsf{IN}(\mu, \Omega), \quad \mu = \mu_i, \Omega = \omega_{ij}, i, j = 1, 2 \tag{13}$$

Conditional distribution $y_t | z_t$:

$$y_t | z_t \sim \mathsf{IN}(\alpha + \beta z_t, \sigma^2) \tag{14}$$

where $\beta = \omega_{12}/\omega_{22}$ and $\alpha = \mu_1 - \beta\mu_2$ and $\sigma^2 = \omega_{11} - \omega_{12}^2/\omega_{22}$

Denote the error terms:

$$u_{1,t} = y_t - E(y_t | z_t) \text{ and } u_{2,t} = y_t - E(y_t) \tag{15}$$

$$v_{1,t} = z_t - E(z_t | y_t) \text{ and } v_{2,t} = z_t - E(z_t) \tag{16}$$

Conditional model:

$$y_t = \alpha + \beta z_t + u_{1,t} \tag{17}$$

Marginal model:

$$z_t = \mu_2 + v_{2,t} \tag{18}$$

where $u_{1,t} \sim \mathsf{IN}(0, \sigma^2)$ and $v_{2,t} \sim \mathsf{IN}(0, \omega_{22})$

- z_t is weakly exogenous for parameters in conditional model (α, β, σ^2) as the parameters in the conditional and marginal models are variation free.
- Arbitrary choices of μ_2, ω_{22} do not constrain the parameters in the conditional model.

Consider alternative conditioning:

$$z_t = \gamma + \delta y_t + \nu_{1,t} \quad (19)$$

and marginal:

$$y_t = \mu_1 + u_{2,t} \quad (20)$$

where $\delta = \omega_{12}/\omega_{11}$, $\gamma = \mu_2 - \delta\mu_2$, and
$V(\nu_{1,t}) = \tau^2 = \omega_{22} - \omega_{21}^2/\omega_{11}$.

- Now y_t is weakly exogenous for δ, γ, τ^2.
- Weak exogeneity is defined relative to parameters of interest (can they be obtained from the conditional model alone).

Predeterminedness does not imply WE or vice versa:

- Regardless of parameter of interest, by construction y_t is predetermined in conditional model of z_t (and vice versa). Cov$(z_t, u_{1,t}) = 0$:

$$
\begin{aligned}
Cov(z_t, u_{1,t}) &= Cov(z_t, y_t - E[y_t|z_t]) \\
&= Cov(z_t, y_t) + Cov(z_t, -E[y_t|z_t]) \\
&= \omega_{12} + Cov(z_t, -\mu_1 - \beta(z_t - \mu_2)) \\
&= \omega_{12} - \beta Var(z_t) \\
&= \omega_{12} - \frac{\omega_{12}}{\omega_{22}}\omega_{22}
\end{aligned}
$$

- But: e.g. when $\gamma = \mu_2 - \delta\mu_2$ is the parameter of interest in cond. model of y_t, then z_t is predetermined, but not weakly exogenous.

Here weak-exogeneity cannot be tested. Not the case in more complex models (particularly useful in cointegrated VARs).

Exogeneity in a Cobweb Model (control rule to control y):

$$y_t = z_t\beta + \epsilon_{y,t} \quad (21)$$

$$z_t = z_{t-1}\delta_1 + y_{t-1}\delta_2 + \epsilon_{z,t} \quad (22)$$

$$\epsilon_t = \begin{bmatrix} \epsilon_{y,t} \\ \epsilon_{z,t} \end{bmatrix} \sim \mathsf{IN}\left(0, \Sigma\right), \qquad \Sigma = \begin{pmatrix} \sigma_{yy} & \sigma_{yz} \\ \sigma_{yz} & \sigma_{zz} \end{pmatrix} \quad (23)$$

Model is a restricted factorisation of a VAR of y_t, z_t, where the unrestricted reduced form is given by:

$$y_t = y_{t-1}\beta\delta_2 + z_{t-1}\beta\delta_1 + v_t \quad (24)$$

$$z_t = z_{t-1}\delta_1 + y_{t-1}\delta_2 + \epsilon_{z,t} \quad (25)$$

with:

$$\begin{bmatrix} v_t \\ \epsilon_{z,t} \end{bmatrix} \sim \mathsf{IN}\left(0, \Omega\right), \quad \Omega = \begin{pmatrix} \sigma_{yy} + 2\beta\sigma_{yz} + \beta^2\sigma_{zz} & \sigma_{yz} + \beta\sigma_{zz} \\ \sigma_{yz} + \beta\sigma_{zz} & \sigma_{zz} \end{pmatrix}$$

The VAR can be partitioned into a **conditional** and a **marginal** model:

$$y_t = z_t b + y_{t-1} a_1 + z_{t-1} a_2 + u_t \quad (26)$$

$$z_t = z_{t-1} \delta_1 + y_{t-1} \delta_2 + \epsilon_{z,t} \quad (27)$$

where $u_t \sim \mathsf{IN}(0, \sigma^2)$.

The coefficients in (26) map to cobweb model (21) such that

- $b = \beta + \frac{\sigma_{yz}}{\sigma_{zz}}$
- $a_i = \delta_i \frac{\sigma_{yz}}{\sigma_{zz}}$ where $i = [1, 2]$.

If $\sigma_{yz} = 0$, then z_t is WE for the parameter of interest β.

If $\sigma_{yz} = 0$:

- z is 'predetermined' in (21) and z is also weakly exogenous for β

However, if $\sigma_{yz} \neq 0$ then:

- z is not predetermined in model (21) as z_t is non-orthogonal to the error $\epsilon_{y,t}$
- while z it is predetermined in the conditional model (26), however, then z not weakly exogenous for β which cannot be recovered from the conditional model alone.

Pre-determinedness does not imply weak-exogeneity and valid conditioning.

Exogeneity in a Cointegrated VAR

$$y_t = \sum_{j=1}^{s} A_j y_{t-j} + \mu + \epsilon_t \tag{28}$$

$$\epsilon_t \sim \text{IN}\left(0, \Sigma\right), \text{ with } \Sigma = \begin{pmatrix} \Sigma_{xx} & \Sigma_{xz} \\ \Sigma_{xz} & \Sigma_{zz} \end{pmatrix} \tag{29}$$

(30)

Consider $s = 2$ and assume reduced rank$(\Pi) = 1$ where ($\Pi = \alpha\beta'$)

$$\Delta y_t = \alpha\beta' y_{t-1} + \Gamma \Delta y_{t-1} + \mu + \epsilon_t \tag{31}$$

where $y_t = (x_t, z_t)'$, $\epsilon_t = (\epsilon_{x,t}, \epsilon_{z,t})'$ and $\Delta y_t = y_t - y_{t-1}$.

Can write out the full CVAR system as:

$$\begin{bmatrix} \Delta x_t \\ \Delta z_t \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \begin{bmatrix} \beta_1 & \beta_2 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \begin{bmatrix} \Delta x_{t-1} \\ \Delta z_{t-1} \end{bmatrix} + \begin{bmatrix} \mu_x \\ \mu_z \end{bmatrix} + \begin{bmatrix} \epsilon_{x,t} \\ \epsilon_{z,t} \end{bmatrix}$$

Cointegrating vector (equilibrium relationship) given by:

$$\beta' y_t = \beta_1 x_t + \beta_2 z_t \tag{32}$$

Re-write VAR as conditional and marginal:

From the joint CVAR model define $m_{x,t}$ and $m_{z,t}$ as follows:

$$m_{x,t} = \mu_x + \alpha_1 \beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} \quad (33)$$

$$m_{z,t} = \mu_z + \alpha_2 \beta' y_{t-1} + \Gamma_2 \Delta y_{t-1} \quad (34)$$

Using the multivariate normal distribution of ϵ_t, the conditional expectation of Δx_t given $\Delta z_t, y_{t-1}$ is equal to:

$$E[\Delta x_t | \Delta z_t, y_{t-1}] = m_{x,t} + D(\Delta z_t - m_{z,t}) \quad (35)$$

where $D = \Sigma_{xz}\Sigma_{zz}^{-1}$. Substituting for $m_{x,t}, m_{z,t}$ in (35) and collecting terms yields the resulting conditional model.

Conditional Model:

$$\begin{aligned}
\Delta x_t &= (\mu_x - D\mu_z) + (\alpha_1 - D\alpha_2)\beta' y_{t-1} + D\Delta z_t + (\Gamma_1 - D\Gamma_2)\Delta y_{t-1} + \nu_{x,t} \\
&= (\mu_x - D\mu_z) + \beta_1(\alpha_1 - D\alpha_2)x_{t-1} + \beta_2(\alpha_1 - D\alpha_2)z_{t-1} + D\Delta z_t + \\
&\quad (\Gamma_{11} - D\Gamma_{21})\Delta x_{t-1} + (\Gamma_{12} - D\Gamma_{22})\Delta z_{t-1} + \nu_{x,t} \\
&= \gamma_0 + \gamma_1 x_{t-1} + \gamma_2 z_{t-1} + \gamma_3 \Delta z_t + \gamma_4 \Delta x_{t-1} + \gamma_5 \Delta z_{t-1} + \nu_{x,t}
\end{aligned}$$

Where $D = \Sigma_{xz}\Sigma_{zz}^{-1}$, $\Gamma_i = (\Gamma_{i1}, \Gamma_{i2})$, and $\nu_{x,t} = \epsilon_{x,t} - D\epsilon_{z,t}$.

Conditional Model:

$$
\begin{aligned}
\Delta x_t &= (\mu_x - D\mu_z) + (\alpha_1 - D\alpha_2)\beta' y_{t-1} + D\Delta z_t + (\Gamma_1 - D\Gamma_2)\Delta y_{t-1} + \nu_{x,t} \\
&= (\mu_x - D\mu_z) + \beta_1(\alpha_1 - D\alpha_2)x_{t-1} + \beta_2(\alpha_1 - D\alpha_2)z_{t-1} + D\Delta z_t + \\
&\quad (\Gamma_{11} - D\Gamma_{21})\Delta x_{t-1} + (\Gamma_{12} - D\Gamma_{22})\Delta z_{t-1} + \nu_{x,t} \\
&= \gamma_0 + \gamma_1 x_{t-1} + \gamma_2 z_{t-1} + \gamma_3 \Delta z_t + \gamma_4 \Delta x_{t-1} + \gamma_5 \Delta z_{t-1} + \nu_{x,t}
\end{aligned}
$$

Where $D = \Sigma_{xz}\Sigma_{zz}^{-1}$, $\Gamma_i = (\Gamma_{i1}, \Gamma_{i2})$, and $\nu_{x,t} = \epsilon_{x,t} - D\epsilon_{z,t}$.

Marginal model:

$$
\begin{aligned}
\Delta z_t &= \mu_z + \alpha_2\beta' y_{t-1} + \Gamma_2 \Delta y_{t-1} + \epsilon_{z,t} \\
&= \mu_z + (\alpha_2\beta_1)x_{t-1} + (\alpha_2\beta_2)z_{t-1} + \Gamma_{21}\Delta x_{t-1} + \Gamma_{22}\Delta z_{t-1} + \epsilon_{z,t} \\
&= \psi_0 + \psi_1 x_{t-1} + \psi_2 z_{t-1} + \psi_3 \Delta x_{t-1} + \psi_4 \Delta z_{t-1} + \epsilon_{z,t}
\end{aligned}
$$

Weak Exogeneity in the CVAR:

If β (the long-run equilibrium) is the parameter of interest, then direct test for weak exogeneity is testing α_i adjustment coefficients.

- z_t is weakly exogenous for β if $\alpha_2 = 0$
- x_t is weakly exogenous for β if $\alpha_1 = 0$

Testable: LR tests, (or bootstrap).

Note: both series cannot be weakly exogenous simultaneously, as otherwise there is no cointegration.

If $\mathbf{\Gamma}$ are parameters of interest, then weak exogeneity requires restrictions on covariance matrix Σ.

Strong Exogeneity in the CVAR:

- x_t will not Granger-cause z_t if and only if $\Gamma_{21} = 0$ and $(\alpha_2 \beta_1) = 0$.
- z_t will not Granger-cause x_t if and only if $\Gamma_{12} = 0$ and $(\alpha_1 \beta_2) = 0$.

If $\beta_1, \beta_2 \neq 0$ then granger non-causality is a sufficient condition for strong exogeneity (which add. requires weak exogeneity $\alpha = 0$) in the 1-lag, rank=1, CVAR example.

Consider two cointegrating vectors?

Super Exogeneity in the CVAR

Engle and Hendry (1993) on Super Exogeneity:

- Weak Exogeneity (testable on α)
- Parameter constancy (testable: recursive or breaks)
- Parameters in cond. invariant to shifts in marginal

Here: **focus on testing invariance** to changes/shifts in marginal density.

Shock in marginal process – does this change how variable reacts in conditional model?

Example: Policy intervention in marginal model – increase taxes, emission standards etc. Response in conditional: does variable respond the way we expect given the estimated model?

$\rightarrow$ Makes the '**Lucas Critique**' testable.

Lucas (1976) criticised using econometric models for policy analysis: implementing policy would alter the structure the model was attempting to capture.

- Agents: form model-based expectations about z when making decisions about y, then λ_1 depends on λ_2, and λ_1 will change if policy alters λ_2.
- This may be the case even if weak exogeneity holds.

Lucas critique focuses on two model properties:
Parameter constancy and **invariance**: Super exogeneity provides a concept to test what Lucas criticised.

Hendry 1988, and Engle and Hendry 1993:

- **A)** Test for constancy of λ_1 and λ_2. If λ_1 is constant, but λ_2 is not, then λ_1 is invariant to the interventions that ocurred – and Lucas critique can not apply.
- **B)** Develop marginal model until its parameters are empirically constant – model how λ_2 varies over time by including dummies, etc. Then test the significance of these dummies in the conditional model. Insignificance in conditional model demonstrates invariance of λ_1.

$\rightarrow$ **empirical presence of super-exogeneity (testable) refutes Lucas critique in practise.**

Testing Invariance:

Develop marginal model until its parameters are empirically constant – model how λ_2 varies over time by including dummies (known shocks/interventions).

Then test the significance of these dummies in the conditional model. Insignificance in conditional model demonstrates invariance of λ_1.

Unknown unknowns – detect shocks in marginal (how?) and subsequently test if they are already accounted for in the parameters of the conditional.

Parameter invariance essential in policy models: else mis-predict under regime shifts.

Super exogeneity combines parameter invariance with valid conditioning so crucial for economic policy.

- Automatic test in Hendry and Santos (2010) & Castle, Hendry, Martinez (2016): detect shocks in marginal models, retain all significant outcomes and test their relevance in conditional model.
- No *ex ante* knowledge of timing or magnitudes of breaks: need not know DGP of marginal variables.
- Test has correct size under null of super exogeneity for a range of sizes of marginal-model saturation tests. **Power to detect failures of super exogeneity when location shifts in marginal models:applies equally to models with expectations like NKPC.**

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First stage is detection of shocks in marginal,
retaining dummies at significance level α_1, here m are detected:

$$\mathbf{z}_t = \boldsymbol{\pi}_0 + \sum_{j=1}^{s} \boldsymbol{\Pi}_j \mathbf{x}_{t-j} + \sum_{i=1}^{m} \boldsymbol{\rho}_{i,\alpha_1} 1_{\{t=t_i\}} + \mathbf{v}^*_{2,t} \tag{36}$$

Second stage adds m retained indicators to conditional:

$$y_t = \mu_0 + \boldsymbol{\beta}' \mathbf{z}_t + \sum_{i=1}^{m} \tau_{i,\alpha_2} 1_{\{t=t_i\}} + \epsilon_t \tag{37}$$

Conduct F-test for significance of $(\tau_{1,\alpha_2} \ldots \tau_{m,\alpha_2})$ at level α_2.

Test has power as significant impulse indicators capture outliers not explained by regressors.

The invariance test involves two stages.

A] IIS and/or SIS is applied to marginal system for all current conditioning variables $\mathbf{z}_t$: significant indicators are recorded.

B] Those indicators then tested for significance in the conditional equation $(y_t|\mathbf{z}_t, \cdots)$

Retaining s lags of all variables $\mathbf{x}_t = (\mathbf{y}_t : \mathbf{z}_t)'$, SIS is applied at significance level α_1 leading to the selection of m step indicators:

$$\mathbf{z}_t = \psi_0 + \sum_{i=1}^{s} \mathbf{\Psi}_i \mathbf{x}_{t-i} + \sum_{j=1}^{m} \eta_{j,\alpha_1} 1_{\{t \leqslant T_j\}} + \mathbf{v}_{2,t} \qquad (38)$$

where (38) is selected to be congruent. Significant step-indicator coefficients are denoted η_{j,α_1} to emphasize dependence on α_1.

Simulating *Autometrics*, Castle, Doornik, Hendry, and Pretis (2015) show the gauge, g, of SIS in (38) is close to α_1; and SIS has potency for detecting substantive location shifts close to power for known shifts.

The m significant step indicators $\{1_{\{t \leqslant j\}}\}$ are retained in (38) when:

$$\left|t_{\widehat{\eta}_{i,\alpha_1}}\right| > c_{\alpha_1} \tag{39}$$

when c_{α_1} is the critical value for α_1.
Collect the indicators at each t in an m vector $\mathbf{d}_t$, then (for $s = 1$):

$$y_t = \gamma_0 + \boldsymbol{\gamma}_1'\mathbf{z}_t + \boldsymbol{\pi}'\mathbf{x}_{t-1} + \boldsymbol{\delta}'\mathbf{d}_t + \epsilon_t \tag{40}$$

where $\boldsymbol{\delta} = \left(\delta'_{1,\alpha_1} \ldots \delta'_{m,\alpha_1}\right)' = \mathbf{0}$ under the null, to be tested as an added-variable set in the conditional equation (40) **without selection**.
Use the $\mathsf{F}_{\mathsf{Inv}(\boldsymbol{\delta}=\mathbf{0})}$-test, at significance level α_2 which rejects when:

$$\mathsf{F}_{\mathsf{Inv}(\boldsymbol{\delta}=\mathbf{0})} > c_{\alpha_2}. \tag{41}$$

The simulation DGP is:

$$\begin{pmatrix} y_t \\ z_t \end{pmatrix} \mid \mathbf{x}_{t-1} \sim \mathsf{IN}_2 \left[\begin{pmatrix} \gamma_1 + \rho\gamma_{2,t} \\ \gamma_{2,t} \end{pmatrix}, \sigma_{22} \begin{pmatrix} \sigma_{22}^{-1}\sigma_{11} + \rho^2\theta_{(t)} & \rho\theta_{(t)} \\ \rho\theta_{(t)} & \theta_{(t)} \end{pmatrix} \right]$$

$$\mathsf{E}[y_t | z_t, x_{t-1}] = \gamma_1 + \rho\gamma_{2,t} + \frac{\rho\sigma_{22}\theta_{(t)}}{\sigma_{22}\theta_{(t)}}(z_t - \gamma_{2,t}) = \gamma_1 + \rho z_t$$

$\gamma_{2,t} = 1 + \lambda 1_{\{t \leqslant T_1\}}$ and $\theta_{(t)} = 1 + \theta 1_{\{t \leqslant T_2\}}$ (invariance still holds)

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The simulation DGP is:

$$\begin{pmatrix} y_t \\ z_t \end{pmatrix} \mid \mathbf{x}_{t-1} \sim \mathsf{IN}_2 \left[\begin{pmatrix} \gamma_1 + \rho\gamma_{2,t} \\ \gamma_{2,t} \end{pmatrix}, \sigma_{22} \begin{pmatrix} \sigma_{22}^{-1}\sigma_{11} + \rho^2\theta_{(t)} & \rho\theta_{(t)} \\ \rho\theta_{(t)} & \theta_{(t)} \end{pmatrix} \right]$$

$$\mathsf{E}[y_t|z_t, x_{t-1}] = \gamma_1 + \rho\gamma_{2,t} + \frac{\rho\sigma_{22}\theta_{(t)}}{\sigma_{22}\theta_{(t)}}(z_t - \gamma_{2,t}) = \gamma_1 + \rho z_t$$

$\gamma_{2,t} = 1 + \lambda 1_{\{t \leqslant T_1\}}$ and $\theta_{(t)} = 1 + \theta 1_{\{t \leqslant T_2\}}$ (invariance still holds)

Three different null states need to be investigated:
(a) a constant marginal distribution for z_t;
(b) a location shift in that distribution (so $\lambda \neq 0$); and
(c) a variance shift (so $\theta \neq 0$).

SIS-based tests at stage 1 should not use too tight an α_1 or may find no indicators, so $\mathsf{F}_{\mathsf{Inv}(\boldsymbol{\delta}=\mathbf{0})}$ would have a zero null rejection frequency.

Outcomes at α_1=0.01, s = 4 in the GUM

Constant marginal process	$T = 50$	$T = 100$	$T = 200$
Stage 1 gauge: $\alpha_1 = 0.01$	0.035	0.033	0.044
Stage 2 NRF: $\alpha_2 = 0.01$	**0.006**	**0.009**	**0.009**

Constant marginal process	$T = 50$	$T = 100$	$T = 200$
Stage 1 gauge: $\alpha_1 = 0.01$	0.035	0.033	0.044
Stage 2 NRF: $\alpha_2 = 0.01$	**0.006**	**0.009**	**0.009**

Location shift in z_t		$\lambda = 2$			$\lambda = 10$	
	$T = 50$	$T = 100$	$T = 200$	$T = 50$	$T = 100$	$T = 200$
$\alpha_1 = 0.01$						
Stage 1 gauge	0.034	0.027	0.043	0.018	0.018	0.035
Stage 1 potency	0.191	0.186	0.205	**0.957**	**0.962**	**0.965**
$\alpha_2 = 0.01$						
Stage 2 NRF	**0.009**	**0.010**	**0.011**	**0.010**	**0.009**	**0.010**

Constant marginal process	T = 50	T = 100	T = 200
Stage 1 gauge: $\alpha_1 = 0.01$	0.035	0.033	0.044
Stage 2 NRF: $\alpha_2 = 0.01$	**0.006**	**0.009**	**0.009**

Location shift in z_t	$\lambda = 2$			$\lambda = 10$		
	T = 50	T = 100	T = 200	T = 50	T = 100	T = 200
$\alpha_1 = 0.01$						
Stage 1 gauge	0.034	0.027	0.043	0.018	0.018	0.035
Stage 1 potency	0.191	0.186	0.205	**0.957**	**0.962**	**0.965**
$\alpha_2 = 0.01$						
Stage 2 NRF	**0.009**	**0.010**	**0.011**	**0.010**	**0.009**	**0.010**

Variance shift in z_t	$\theta = 2$			$\theta = 10$		
	T = 50	T = 100	T = 200	T = 50	T = 100	T = 200
$\alpha_1 = 0.01$						
Stage 1 gauge	0.042	0.051	0.067	0.060	0.083	0.113
Stage 1 potency	0.030	0.030	0.035	0.041	0.043	0.071
$\alpha_2 = 0.01$						
Stage 2 NRF	0.006	0.008	0.009	0.006	0.009	0.010

The DGP for violations of invariance (here due to a failure of weak exogeneity under non-constancy):

$$\begin{pmatrix} y_t \\ z_t \end{pmatrix} \sim \mathsf{IN}_2 \left[\begin{pmatrix} \gamma_1 + \rho\gamma_{2,t} \\ \gamma_{2,t} \end{pmatrix}, \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} \right] \quad (42)$$

so letting $\sigma_{12}/\sigma_{22} = \beta$ leads to the conditional relation:

$$\mathsf{E}\left[y_t \mid z_t\right] = \gamma_1 + (\rho - \beta)\gamma_{2,t} + \beta z_t \quad (43)$$

$$\gamma_{2_t} = \lambda 1_{\{t > T_1\}} \quad (44)$$

$d = \lambda/\sqrt{\sigma_{22}} : \{1, 2, 2.5, 3, 4\}$,
$\beta : \{0.75, 1, 1.5, 1.75\}$ (reducing departure from WE, $\rho = 2$)

The DGP location shift: $T_s = 80 - 100$.
$M = 1,000$ replications at $\alpha_1 = \alpha_2 = 0.01$.

Table 6: Stage 1 gauge and potency for $\beta = 1$.

d	1	2	2.5	3	4
Stage 1 gauge:	0.040	0.028	0.026	0.023	0.021
Stage 1 potency:	0.223	0.575	0.737	0.813	0.930

The procedure is over-gauged, but this is not crucial at stage 1.

Potency rises rapidly with d (magnitude of shift).

When correct step indicator is always included in the conditional model, powers approximate the maximum achievable using $\alpha_2 = 0.01$.

Table 7: Power using a known step indicator.

$d : \beta$	0.75	1	1.5	1.75
1	1.00	1.00	0.886	**0.270**
2	1.00	1.00	1.000	0.768
2.5	1.00	1.00	1.000	0.855
3	1.00	1.00	1.000	0.879
4	1.00	1.00	1.000	**0.931**

For large magnitudes of d or substantive departures from weak exogeneity, $\rho - \beta$, invariance almost always rejected,
but power falls rapidly when both the location shift magnitude declines and the weak exogeneity violation disappears.

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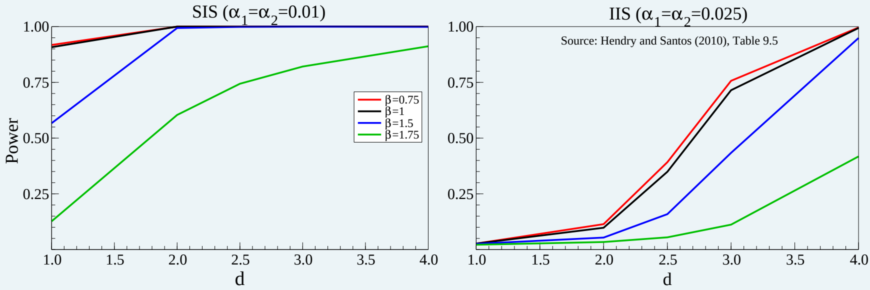
Table 8: Stage 2 power for a failure of invariance, (for $\rho = 2$)

d : β	0.75	1	1.5	1.75
1	0.918	0.908	0.567	**0.127**
2	1.000	1.000	0.994	0.604
2.5	1.000	1.000	0.999	0.744
3	1.000	1.000	1.000	0.821
4	1.000	1.000	0.999	**0.912**

Power of the SIS test is close to the optimal test power in Table 7, despite not knowing the number or timing of any shifts.

Figure 2


SIS (α1=α2=0.01)
IIS (α1=α2=0.025)
Source: Hendry and Santos (2010), Table 9.5
β=0.75
β=1
β=1.5
β=1.75
Power
1.00
0.75
0.50
0.25
1.0
1.5
2.0
2.5
3.0
3.5
4.0
d

Exogeneity in Climate Econometrics – see Pretis (2017)

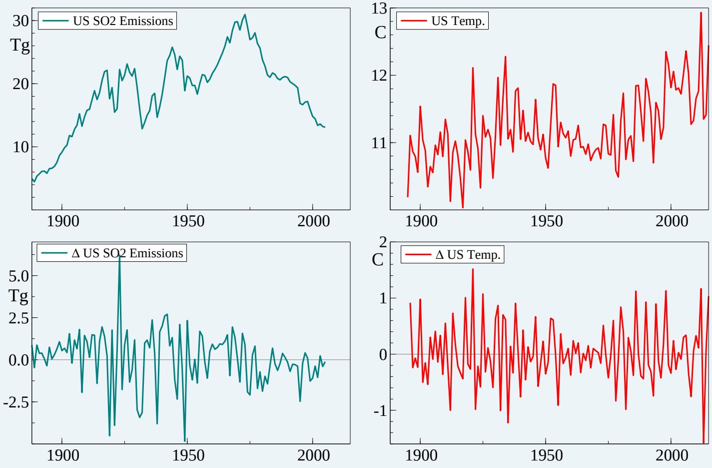
Empirical studies on:

- Impacts of climate (temperature changes etc.) onto economic activity
- Impacts of economic activity onto climate

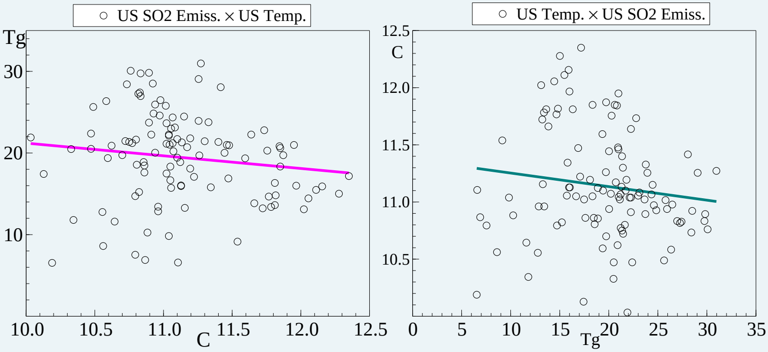
Climate economic system: where each side considers conditional models.

One economic variable (e_t) and one climate variable (c_t)

US SO_2 Emissions (e_t) **US Temp.** (c_t)



$$\underbrace{\hat{e}_t = \underset{(11.69)}{43.65} - \underset{(1.05)}{2.15}\, c_t}_{\text{Econ. cond. on Climate}} \quad \textbf{vs.} \quad \underbrace{\hat{c}_t = \underset{(0.17)}{11.49} - \underset{(0.008)}{0.018}\, e_t}_{\text{Climate cond. on Econ.}}$$



Climate-Economic System: **DGP** is Joint Density of $y_t = (e_t' : c_t')$

- e_t socio-economic processes
- c_t climate processes

Data Generating Process (DGP):

$$D_Y(Y_T^1 | Y_0, \zeta) = \prod_{t=1}^{T} D_y(y_t | Y_{t-1}, \zeta_t)$$

Model:

$$f_Y(Y_T^1 | Y_0, \theta) = \prod_{t=1}^{T} f_y(y_t | Y_{t-1}, \theta)$$

Joint Model for $y_t = (e_t, c_t)$ as a VAR:

$$y_t = \sum_{j=1}^{s=2} A_j y_{t-j} + \mu + \epsilon_t \qquad \epsilon_t \sim \mathsf{IN}\left(0, \Sigma\right), \text{ with } \Sigma = \begin{pmatrix} \Sigma_e & \Sigma_{ec} \\ \Sigma_{ec} & \Sigma_c \end{pmatrix}$$

Joint Model for $y_t = (e_t, c_t)$ as a VAR:

$$y_t = \sum_{j=1}^{s=2} A_j y_{t-j} + \mu + \epsilon_t \qquad \epsilon_t \sim \mathsf{IN}(0, \Sigma), \text{ with } \Sigma = \begin{pmatrix} \Sigma_e & \Sigma_{ec} \\ \Sigma_{ec} & \Sigma_c \end{pmatrix}$$

Cointegration Tests for e_t, c_t (US Temp. & US SO_2 emissions)

Rank	Trace Test	Trace Test (Bartlett)	Bootstrap Test
0	$p<0.001$	$p<0.001$	$p<0.001$
1	$p=0.012$	$p=0.013$	$p=0.075$

Sample: 1897 - 2005, T=109

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Climate-Economic System as a CVAR :

$$\Delta y_t = \alpha\beta' y_{t-1} + \Gamma \Delta y_{t-1} + \mu + \epsilon_t$$

Climate-Economic System as a CVAR :

$$\Delta y_t = \alpha\beta' y_{t-1} + \Gamma\Delta y_{t-1} + \mu + \epsilon_t$$

$$\begin{bmatrix}\Delta e_t \\ \Delta c_t\end{bmatrix} = \begin{bmatrix}\alpha_1 \\ \alpha_2\end{bmatrix}\begin{bmatrix}\beta_1 & \beta_2\end{bmatrix}\begin{bmatrix}e_{t-1} \\ c_{t-1}\end{bmatrix} + \begin{bmatrix}\Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22}\end{bmatrix}\begin{bmatrix}\Delta e_{t-1} \\ \Delta c_{t-1}\end{bmatrix} + \begin{bmatrix}\mu_e \\ \mu_c\end{bmatrix} + \begin{bmatrix}\epsilon_{e,t} \\ \epsilon_{c,t}\end{bmatrix}$$

$$\begin{bmatrix}\Delta \hat{e}_t \\ \Delta \hat{c}_t\end{bmatrix} = \underbrace{\begin{bmatrix}\underset{(0.005)}{0.003} \\ \underset{(0.001)}{-0.008}\end{bmatrix}}_{\hat{\alpha}}\underbrace{\begin{bmatrix}1 & \underset{(14.27)}{78.8}\end{bmatrix}}_{\hat{\beta}'}\begin{bmatrix}e_{t-1} \\ c_{t-1}\end{bmatrix} + \underbrace{\begin{bmatrix}\underset{(0.1)}{-0.014} & \underset{(0.36)}{-0.67} \\ \underset{(0.03)}{-0.02} & \underset{(-0.07)}{0.1}\end{bmatrix}}_{\hat{\Gamma}}\begin{bmatrix}\Delta e_{t-1} \\ \Delta c_{t-1}\end{bmatrix} + \underbrace{\begin{bmatrix}\underset{(4.85)}{-2.07} \\ \underset{(1.31)}{6.91}\end{bmatrix}}_{\hat{\mu}}$$

$$\begin{aligned} T &= 109,\quad n = 8,\quad \text{LogLik} = -264.69,\quad F_{ar}(8, 200) = 2.36[p = 0.02], \\ \chi^2_{norm}(4) &= 10.06[p = 0.04],\quad F_{het}(18, 283) = 0.947[p = 0.52] \end{aligned}$$

Model Fit

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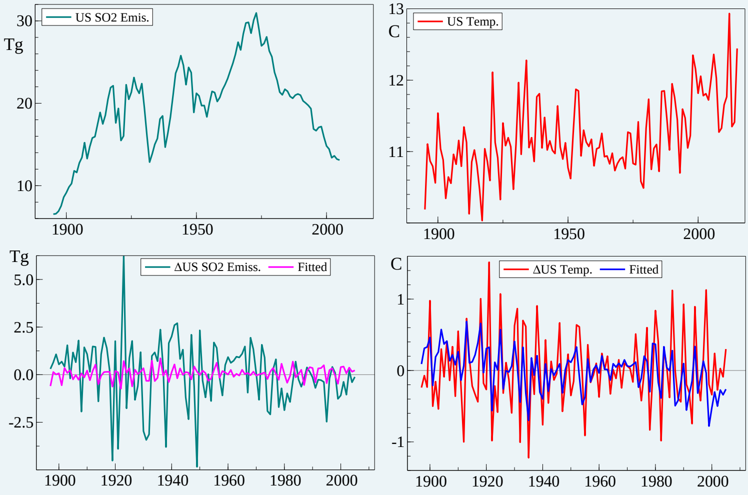
US SO_2 Emissions (e_t) | US Temp. (c_t)



Simple bi-varate system, however, joint model often difficult to model.
Partition the joint density $f_y(y_t|Y_{t-1},\theta)$ into conditional and marginal:

Empirical Climate Impacts

$$f_y(y_t|Y_{t-1},\theta) = \underbrace{f_{e|c}(e_t|c_t, Y_{t-1}, \lambda_{e|c})}_{\text{Conditional}} \cdot \underbrace{f_c(c_t|Y_{t-1},\lambda_c)}_{\text{Marginal}}$$

Empirical Climate Model

$$f_y(y_t|Y_{t-1},\theta) = f_{c|e}(c_t|e_t, Y_{t-1}, \phi_{c|e}) \cdot f_e(e_t|Y_{t-1},\phi_e)$$

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Empirical Climate Model

$$f_y(y_t|Y_{t-1},\theta) = f_{c|e}(c_t|e_t, Y_{t-1}, \phi_{c|e}) \cdot f_e(e_t|Y_{t-1}, \phi_e)$$

Conditions to study the conditional model alone to:

(i) conduct inference on the parameter of interest (weak exogeneity)
(ii) conduct conditional forecasts (strong exogeneity)
(iii) counterfactual policy analysis (super-exogeneity)

Inference in the conditional model alone?

$$f_y(y_t|Y_{t-1},\theta) = \underbrace{f_{e|c}(e_t|c_t, Y_{t-1}, \lambda_{e|c})}_{\text{Conditional}} \cdot \underbrace{f_c(c_t|Y_{t-1}, \lambda_c)}_{\text{Marginal}}$$

Weak Exogeneity:

- $\psi = \psi(\lambda_{e|c})$, parameter of interest a function of $\lambda_{e|c}$ alone, and $\lambda_{e|c}$ & λ_c are variation free.
- Cointegration: Physical interpretation for adjustment α (Weak Ex.)
- Weak Ex. cannot hold for both variables e_t, c_t for β

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	Emp. Climate Impacts	Emp. Climate Model
Restriction	$\alpha_1 = 0$ (Emiss.)	$\alpha_2 = 0$ (Temp.)
LR	$\chi^2(1)$ = 0.26 [p=0.61]	$\chi^2(1)$ = 19.55 [p$<$0.00]
Bootstrap	0.25 [p=0.66]	19.96 [p$<$0.00]

Temp. adjusts to coint. relation: β enters marginal model of c_t

Granger Non-Causality: **Climate Taking** vs. **Climate Setting**

- **Climate-Taking** if: $f_c(c_t|Y_{t-1}, \lambda_c) = f_c(c_t|c_{t-1}, \lambda_c)$
- **Climate Setting** if: $f_c(c_t|Y_{t-1}, \lambda_c) = f_c(c_t|c_{t-1}, e_{t-1}, \lambda_c)$

- Cond. forecasts of impacts if region is **not** climate-setting.
- Reverse applies to climate forecasts: socio-econ/policy adapting to climate observations (e.g. COP21 Paris)

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- Cond. forecasts of impacts if region is **not** climate-setting.
- Reverse applies to climate forecasts: socio-econ/policy adapting to climate observations (e.g. COP21 Paris)

	Emp. Climate Impacts	**Emp. Climate Model**
Restriction	Excl. Temp. in Emiss. ($\alpha_1\beta' = 0$ $\Gamma_{12} = 0$)	Excl. Emiss. in Temp. ($\alpha_2\beta' = 0$ $\Gamma_{21} = 0$)
LR	$\chi^2(2)$=3.98 [p=0.14]	$\chi^2(2)$ 27.76 [p$<$0.00] (Climate setting)

US: No temp. feedback onto emissions & Climate-setting

Weak exogeneity + invariance to shocks:

$$\frac{\partial \lambda_{e|c}}{\partial \lambda_c} = 0 \quad \forall \lambda_c \in C^{\lambda_c}$$

- **Empirical Climate Impacts**: Counter-factuals (e.g. geo-engin.)
- **Empirical Climate Model**: Test for no-tipping elements
  - physical relationship may break down at tipping element
  - H_0(super exogeneity): no tipping element

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  - physical relationship may break down at tipping element
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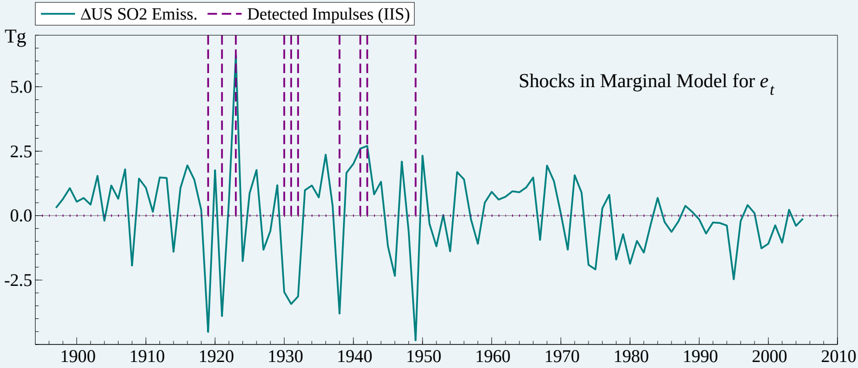
Test in a system by detecting **shocks in marginal model** at $p = \alpha_1$:

$$\Delta e_t = \mu_e + \alpha_1 \beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} + \underbrace{\sum_{i=1}^{T} 1_{i=t} \delta_{i,\alpha_1}}_{\text{All possible Impulses}} + \epsilon_{e,t}$$

Test **joint significance in conditional** climate model at $p = \alpha_2$:

$$\Delta c_t = \lambda_0 + \alpha_2 \beta' y_{t-1} + \lambda_2 \Delta e_{t-1} + \lambda_3 \Delta c_{t-1} + \lambda_4 \Delta e_t + \underbrace{\sum_{i=1}^{m} 1_{t=t_i} \delta_{i,\alpha_1}}_{\text{Detected Impulses}} + \nu_{c,t}$$

Impulse-Indicators (in Marginal Emiss.)	F-Super Exog. (in Temp.)
1919, 1921, 1923, 1930, 1931, 1932, 1938, 1949, 1941, 1942	F(10,94)=1.57 [p=0.13]



Emission shocks already reflected in conditional climate model.

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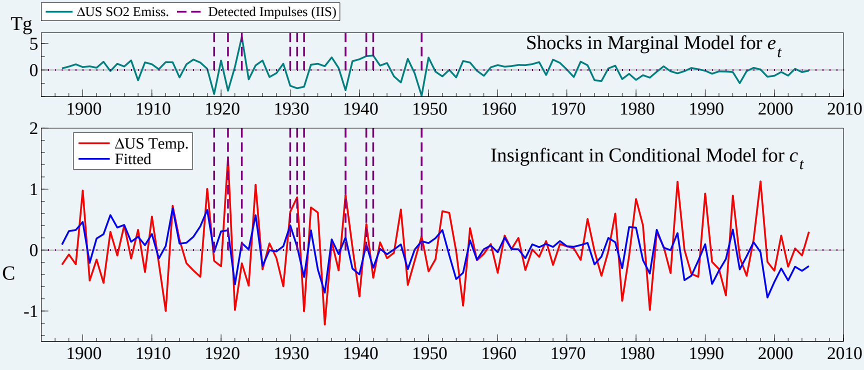

Emission shocks already reflected in conditional climate model.

Conclusions from bivariate system (toy) model:

- Temp. adjusts to Climate-Econ. Equilibrium & Emissions not (Weak Ex.)
- US is Climate Setting for Temp. through SO_2
- No feedback from Temp. onto Emissions (Cond. forecasting)
- Response of Temp to SO_2 invariant to shocks (e.g. WWII) – no tipping point.

Conclusions from bivariate system (toy) model:

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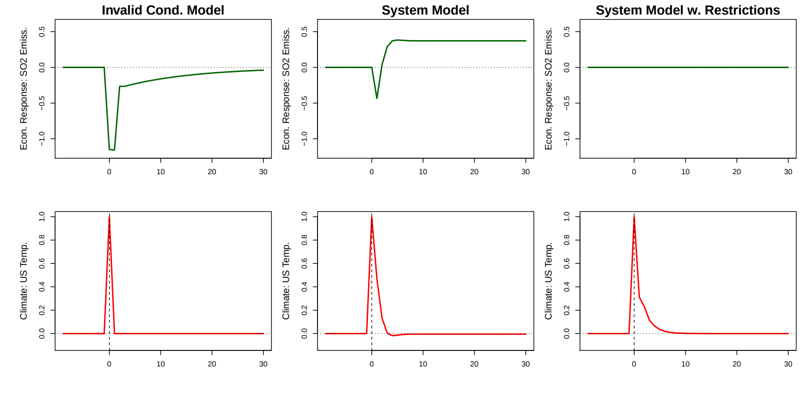
- Response of Temp to SO_2 invariant to shocks (e.g. WWII) – no tipping point.

Contrast to: conclusions of invalid conditional impacts model:

$$\Delta\hat{e}_t = \underset{(5.07)}{6.52} - \underbrace{[\underset{(0.44)}{0.45} c_{t-1} + \underset{(0.03)}{0.07} e_{t-1}]}_{\text{appears significant}} \underbrace{-\underset{(0.33)}{1.15}}_{\hat{\gamma}_3 = \hat{D} = \hat{\Sigma}_{ec}\hat{\Sigma}_c^{-1}} \Delta c_t - \underset{(0.09)}{0.02} \Delta e_{t-1} \underbrace{-\underset{(0.33)}{0.81}}_{\hat{\gamma}_5 = (\hat{\Gamma}_{12} - \hat{D}\hat{\Gamma}_{22})} \Delta c_{t-1}$$

- Incorrectly conclude that $[c_{t-1}, e_{t-1}]$ enter the model (p=0.03) (full system p=0.26)
- Risk of mis-interpreting coeffs. on Δc_t and Δc_{t-1} as impacts

Invalid conditional model vs. system model:



Exogeneity:

- **Weak Ex.**: to study conditional alone
- **Strong Ex.**: Granger Non-Causality + Weak (feedbacks & cond. forecasting)
- **Super Ex.**: Invariance + Weak (policy/counterfactuals/Lucas Critique)

Testable in systems: consider what can be conditioned on.

Predeterminedness does not imply Weak Exogeneity or vice versa.

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