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Testing superexogeneity and invariance in regression models*

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This paper introduces tests of superexogeneity and invariance. Under the null hypothesis the conditional model exhibits parameter constancy, while under the alternative shifts in the process of the independent variables induces shifts in the conditional model. The test is sensitive to particular types of parameter nonconstancy, especially with changing variances and covariances. We relate the test to rational expectations models and the Lucas critique. An empirical example of money demand finds prices and interest rates superexogenous in a conditional model, but when the inflation specification changes, superexogeneity fails although standard specification tests do not.

1. Introduction

Well-specified econometric models should have parameters which are both constant over the historical period used to fit the model and are anticipated to remain constant in the future, thereby allowing *ex ante* forecasting under various scenarios. For example, parameters which measure 'tastes and technology' are sometimes considered to be invariant to changes in policy rules or to shifts in the distributions of other variables taken as given in the model under scrutiny. Conversely, casually-specified regression models may be potentially prone to 'structural breaks' when regimes alter as argued by Lucas (1976). In both cases, tests of claims about invariance or its absence are obviously of interest.

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Empirical evidence on the constancy of parameters over the historical period is typically marshalled by specification tests of the Chow (1960) form or simply by testing for interactions with dummy variables. However, these commonly-used procedures are unsatisfactory if there is no guidance on the number or dates of the breaks which are tested: if incorrect dates are chosen, the test will lack power, and if too many versions are tested, the test size will be far above its nominal value. In contrast, there are almost no procedures to test whether parameters are likely to remain constant in the future or whether they are invariant to changes in regime. This paper presents tests specifically designed to ascertain whether or not parameters have changed *in response* to changes in regime during the historical period. Thus, the tests are closely related to parameter constancy tests, but the causes, periods, and magnitudes of changes to be tested are determined by the economic situation and the historical evidence for regime changes embodied in the conditioning variables. A rejection of the constant parameter conditional model clearly indicates a problem with the hypothesized model, however it is, as always, incorrect to conclude that the alternative model is correct. For example, nonlinearities or nonnormality could be responsible for the rejection and a clever investigator would be advised to test for these directly.

Three distinct concepts are involved when formulating such tests for conditional models: *exogeneity*, *constancy*, and *invariance*, and so we first briefly review these three notions as well as the composite property of *superexogeneity* in section 2. In section 3, the basic formulation of our analysis and relevant definitions are provided. In section 4, the role of expectations and the Lucas Critique are explored for formulating alternatives. In section 5, tests are proposed for invariance of behavioral parameters and superexogeneity of conditioning variables in regression models under various states of nature determined by the degree of nonconstancy in the process generating the observables. Section 6 provides some empirical examples of the proposed tests, and section 7 concludes the paper.

2. Exogeneity, constancy, invariance, and superexogeneity

The joint distribution of y_t and x_t conditional on the sigma field, $\mathscr{F}_t$, consisting of the past of both series and the current and past values of other valid conditioning variables, can be written

$$D_j(y_t, x_t \mid \mathscr{F}_t; \lambda_t) = D_c(y_t \mid x_t, \mathscr{F}_t; \lambda_{1t}) D_m(x_t \mid \mathscr{F}_t; \lambda_{2t}),$$

where D_j, D_c, and D_m refer respectively to the joint density, the conditional density of y given x, and the marginal density of x. The parameters are then labeled correspondingly λ_t, λ_{1t}, and λ_{2t}. This expression recognizes that these parameters may not be constant over time.

Engle, Hendry, and Richard (1983) define a variable x_t as *weakly exogenous* for a set of parameters of interest θ if:

(a) θ is a function of the parameters λ_{1t} *alone*, and
(b) λ_{1t} and the parameters λ_{2t} of the marginal model for x_t are variation-free.

Consequently, if x_t is *weakly exogenous* for θ, there is no loss of information about θ from neglecting to model the process determining x_t. In other words, even perfect knowledge of λ_{2t} could not improve the estimate of λ_{1t} over any period during which they are both constant. If, in addition, y_t fails to Granger cause x_t, then x_t is defined as *strongly exogenous* for θ.

Next, *constancy* of the 'basic' or 'meta' parameters is an essential property of most econometric models. We will be concerned in particular with the constancy of the parameters of the conditional model, λ_{1t}. Good empirical practice checks this assumption, but there is much evidence that parameter variation is common even in final models. See, for example, Judd and Scadding (1982) on U.S. M1 models and Hendry (1979) on 'predictive failure'.

Thirdly, the parameters λ_1 are said to be *invariant* if changes in λ_2 do not imply changes in λ_1. This concept differs from the notion of 'free variation' used for weak exogeneity and a simple example will illustrate the point. Suppose λ_{1t} and λ_{2t} are both scalars and are related by

$$\lambda_{1t} = \phi\lambda_{2t},$$

where ϕ is an unknown scalar. Over periods of constant λ_{2t} there is no information in λ_2 which would be helpful in estimating λ_1, so they are variation-free, however λ_1 clearly is not invariant with respect to λ_2. If instead, the relation were

$$\lambda_1 = \phi_t\lambda_{2t}, \quad \forall t,$$

invariance would be satisfied.

The idea of *invariance* has a long history which includes important contributions from Frisch (1938) (reprinted in 1948) and Haavelmo (1944), both of whom discussed the concept in terms of *autonomy* – the extent to which one relationship remained the same when others altered. In particular, they sought to distinguish the *structural* equations of an economy from the *confluent* relationships induced by the interdependent nature of economic behavior [also see Marschak (1953) and Hurwicz (1962); Aldrich (1989) provides a history]. From a theory perspective, Lucas (1976) criticized invariance claims in 'conventional macro-economic models' on the grounds that agents' expectational mechanisms would alter as policies changed, so that conditional policy simulations would yield misleading inferences in models where the invariant behavioral parameters were not separately estimated from the changing parameters of the expectations processes. Thus, only careful theoretical analysis (such as that now claimed to be associated with 'rational expectations econometrics') could reveal which parameters were potential invariants (sometimes referred to as 'deep' or 'structural' parameters).

In response to such critiques, the theoretical underpinnings of many models have been strengthened, more rigorous testing procedures have been adopted, 'time-varying parameter' processes[1] have been investigated, and so on. Nevertheless, a formal analysis of invariance testing still seems to be lacking. For example, although constancy and invariance are different concepts, tests for the former are sometimes interpreted as tests of the latter. In principle, parameters could vary (over time, say, due to seasonality) but be invariant to policy changes, or be constant over historical interventions but vary with some other alteration in the 'input topology' [see Salmon and Wallis (1982)]. Moreover, the power of constancy tests might be low for investigating invariance claims.

Finally, Engle et al. (1983) defined x_t as *superexogenous* for $\boldsymbol{\theta}$ if x_t is weakly exogenous for $\boldsymbol{\theta}$ and λ_1 is invariant to changes in λ_2. Thus, tests of superexogeneity may be tests of weak exogeneity or tests of invariance or both. As will be seen below, both aspects enter the general formulation. Indeed, tests for invariance raise a number of technical complications due to the need to formulate the alternative model which details how and why the parameters vary. The test must discern whether there has been a shift in regime and, if so, how big a change in parameters would thereby be expected.

3. Formulating superexogeneity and invariance hypotheses

The null hypothesis to be tested in most cases is a linear regression equation which is hypothesized to represent the constant conditional mean of a random variable y_t given another random variable x_t, as well as other information. The tests developed in this paper require careful formulation of the alternative hypotheses which are to be viewed as failures of weak exogeneity, invariance, or superexogeneity.

Consider the joint distribution of two random variables y_t and x_t conditional on an information set $\mathscr{F}_t$ which includes the past of y and x and the current and past of other valid conditioning variables z_t: $\mathscr{F}_t = (\boldsymbol{Y}_{t-1}^1, \boldsymbol{X}_{t-1}^1, \boldsymbol{Z}_t^1)$. We assume throughout that the data generation process of (y_t, x_t) is a conditional normal

[1] However, such processes are not very aptly named since they merely shift which parameters are assumed constant.

distribution (so regressions are linear), given by

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} \bigg| \mathscr{F}_t \sim \mathrm{N}\left[\begin{pmatrix} \mu_t^y \\ \mu_t^x \end{pmatrix}, \begin{pmatrix} \sigma_t^{yy} & \sigma_t^{yx} \\ \sigma_t^{yx} & \sigma_t^{xx} \end{pmatrix}\right] = \mathrm{N}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t), \tag{1}$$

where $\mathrm{N}(\boldsymbol{a}, \boldsymbol{\Omega})$ denotes a normal distribution with mean $\boldsymbol{a}$ and variance matrix $\boldsymbol{\Omega}$. Each of the means and covariances in (1) potentially depends upon the information set $\mathscr{F}_t$, although any particular moments could be constant in practice. The means μ_t^y and μ_t^x are the conditional expectations of y_t and x_t given $\mathscr{F}_t$: $\mu_t^y = \mathrm{E}(y_t \mid \mathscr{F}_t)$ and $\mu_t^x = \mathrm{E}(x_t \mid \mathscr{F}_t)$. Finally, $\boldsymbol{\Sigma}_t$ is the possibly nonconstant error covariance matrix: $\sigma_t^{xx} = \mathrm{E}[(x_t - \mu_t^x)^2 \mid \mathscr{F}_t]$, etc.

The conditional model of interest to the econometrician concerns the relationship between y_t and x_t. From (1), this is in fact given by

$$y_t \mid x_t, \mathscr{F}_t \sim \mathrm{N}[\delta_t(x_t - \mu_t^x) + \mu_t^y, \omega_t], \tag{2}$$

where $\delta_t = \sigma_t^{yx}/\sigma_t^{xx}$ is the regression coefficient of y_t on x_t conditional on $\mathscr{F}_t$, and $\omega_t = \sigma_t^{yy} - (\sigma_t^{yx})^2/\sigma_t^{xx}$ denotes the conditional variance. The analysis generalizes to $\boldsymbol{x}_t$ being a vector, with suitable notation for eq. (7) below, but the bivariate case will suffice to highlight the principles involved.

The parameters of interest in the analysis are β and γ in the theoretical behavioral relationship:

$$\mu_t^y = \beta_t(\boldsymbol{\lambda}_{2t})\mu_t^x + z_t'\gamma, \tag{3}$$

which equation relates the conditional means of y_t and x_t to the set of variables $z_t \in \mathscr{F}_t$. However, we have allowed for the possibility that the 'parameter' β might vary with changes in the parameters of the marginal density of x_t which is denoted by $\boldsymbol{\lambda}_{2t}$, and that the form of this relation may itself be time-varying. In expression (1) it appears that these parameters coincide with μ_t^x and σ_t^{xx}, however more general shifts can easily be considered. Well-known examples of (3) in which β is often taken as constant and invariant are the Permanent Income Hypothesis, where μ_t^y and μ_t^x denote permanent consumption and permanent income, or money demand models in which μ_t^y and μ_t^x become planned money holdings and expected income and z_t characterizes interest rates. In this paper, we maintain the assumption that γ does not vary with $\boldsymbol{\lambda}_{2t}$, since otherwise z_t would merely be reclassified as part of an extended vector $\boldsymbol{x}_t$.

The issue of central concern in this paper is when an econometrician can formulate and estimate the empirical model corresponding to (3) as the regression equation:

$$y_t = \beta x_t + z_t'\gamma + \varepsilon_t \quad \text{where} \quad \varepsilon_t \sim \mathrm{IN}[0, \omega].$$

That step entails exogeneity, constancy, and invariance assumptions which are open to empirical evaluation, and we first analyze these three assumptions and their implications.

Substituting (3) into (2) and rearranging yields

$$y_t \mid x_t, \mathscr{F}_t \sim \mathrm{N}[\beta_t(\lambda_{2t})x_t + z_t'\gamma + \{\delta_t - \beta_t(\lambda_{2t})\}(x_t - \mu_t^x), \omega_t]. \tag{4}$$

As discussed in section 1, three conditions are needed to sustain regression analysis of $y_t \mid x_t, \mathscr{F}_t$:

(a) *Weak exogeneity* of x_t for the parameters of interest. This requires that μ_t^x and σ_t^{xx} do not enter the conditional model. Thus, a necessary condition for the weak exogeneity of x_t for (β, γ) is $\delta_t = \beta_t(\lambda_{2t})$.

(b) *Constancy* of the regression coefficients. The coefficient of x_t in (4) is simply δ_t, so a necessary condition is that $\delta_t = \delta$, $\forall t$. From the definition of δ_t, the ratio of the error covariance to the variance of x_t must be constant over t, and therefore $\omega_t = \omega$, $\forall t$, if $\sigma_t^{yy} = \omega + \delta\sigma_t^{xx}$; otherwise, (5) below would have a heteroskedastic innovation process. We maintain homoskedasticity for simplicity, although generalizations to heteroskedastic processes can be carried out.

(c) *Invariance* of β to the potential changes in λ_{2t}. This requires that $\beta_t(\lambda_{2t}) = \beta_t$, $\forall t$, where the set of parameters β_t may vary over time without depending on variations in λ_{2t}.

Together, these three restrictions entail that $\delta = \beta$. Individually, they constitute necessary conditions to validate constant parameter, invariant conditional models. The conjunction of (a) and (c) ensures that x_t is superexogenous for β, but it is unclear how one might proceed if δ_t nevertheless varied in unknown ways (while maintaining equality to β_t). Consequently, we require all three conditions to hold in practice for the parameters of interest in conditional models. For example, if (a) fails, then generally *both* weak exogeneity and constancy will fail because the mean and variance of x_t appear in the conditional model: hence, changes in μ_t^x and σ_t^{xx} will affect the parameters of the conditional model. Conversely, it is possible that over the historical sample, $D(y_t, x_t \mid \mathscr{F}_t)$ is weakly stationary, so δ is constant and x_t is weakly exogenous for (β, γ) but an intervention alters β post-sample. Thus, (a) and (b) alone do not entail (c).

If (a), (b), and (c) hold, then the conditional distribution (4) becomes

$$y_t \mid x_t, \mathscr{F}_t \sim \mathrm{N}(\beta x_t + z_t'\gamma, \omega), \tag{5}$$

and hence yields the conventional constant parameter regression model. Rewriting (1), given (5), reveals that for this particular model (5) is the condition for the structural error, ε_t, and the reduced form error, η_t, to be uncorrelated in the equations:

$$y_t - x_t\beta - z_t'\gamma = \varepsilon_t \sim \mathrm{IN}[0, \omega], \quad x_t - \mu_t^x = \eta_t \sim \mathrm{IN}[0, \sigma_t^{xx}]. \tag{6}$$

As noted, $\mathrm{E}[\varepsilon_t\eta_t] \neq 0$ may imply nonconstancy as well as invalid exogeneity.

A broad alternative model for a general test of superexogeneity must recognize that, in addition, the behavioral model in (3) may not have constant parameters because β may be affected by λ_{2t}. Specifically, we consider how variations in the *moments* of x might influence β, but maintain that this is a time-invariant relationship. This would allow a class of tests of Lucas Critique assertions to be conducted for historical interventions associated with $\{x_t\}$, together with both constancy tests [of (b) above] and exogeneity tests [of (a)]. Thus, we allow $\beta(\lambda_{2t})$ in (3) to be a function of (μ_t^x, σ_t^{xx}) and approximate $\beta(.)\mu_t^x$ by

$$\beta(\mu_t^x, \sigma_t^{xx})\mu_t^x = \beta_0\mu_t^x + \beta_1(\mu_t^x)^2 + \beta_2\sigma_t^{xx} + \beta_3\sigma_t^{xx}\mu_t^x, \tag{7}$$

assuming $\mu_t^x \neq 0$, $\forall t$. Higher-order expansions could be used if they were thought likely to matter. Given (7), (3) then becomes

$$\mu_t^y = \beta_0\mu_t^x + z_t'\gamma + \beta_1(\mu_t^x)^2 + \beta_2\sigma_t^{xx} + \beta_3\mu_t^x\sigma_t^{xx}, \tag{8}$$

since the term linear in μ_t^x is incorporated through β_0. Under the null of invariance, $\beta_1 = \beta_2 = \beta_3 = 0$ and so $\beta_0 = \beta$.

Substituting (8) into (2) yields

$$\begin{aligned} y_t \mid x_t, \mathscr{F}_t \sim \mathrm{N}[&x_t\beta_0 + z_t'\gamma + (\delta_t - \beta_0)(x_t - \mu_t^x) \\ &+ \beta_1(\mu_t^x)^2 + \beta_2\sigma_t^{xx} + \beta_3\mu_t^x\sigma_t^{xx}, \omega], \end{aligned} \tag{9}$$

so that superexogeneity can fail through the moments of x_t appearing directly in the regression of y_t on x_t even if $\delta_t = \delta$.

An alternative representation is the restricted reduced form, which could remain valid even when x is not weakly exogenous for β. Substituting into (8) and (1) directly:

$$y_t \mid \mathscr{F}_t \sim \mathrm{N}[\mu_t^x\beta_0 + z_t'\gamma + \beta_1(\mu_t^x)^2 + \beta_2\sigma_t^{xx} + \beta_3\mu_t^x\sigma_t^{xx}, \omega],$$

which is operational when an expression for the reduced form of x is available such as $\mu_t^x = \boldsymbol{\pi}_x'\boldsymbol{Z}_t$ for a set of instruments $\boldsymbol{Z}_t$. In this context, the invariance of the parameter β can be tested without the weak exogeneity assumption. Using 2SLS or FIML to estimate the parameters with x_t replacing μ_t^x, it is simple to test various portions of the hypothesis $\beta_1 = \beta_2 = \beta_3 = 0$. In this case, there is no contemporaneous conditioning but it is still possible and interesting to test the hypothesis of invariance.

4. Future expectations, invariance, and the Lucas Critique

Many econometric models derive from economic theories involving expectations. Such models can often be written in precisely the form of (5), except that their parameters are no longer invariant to the process of the forcing variables as was so effectively pointed out by Lucas (1976). Thus tests of backward-looking models are typically conjectured to suffer from parameter nonconstancy and noninvariance. Here we develop a specific class of maintained hypotheses against which such models can be tested. Thus the Lucas Critique can be used to formulate particular alternatives to a backward-looking conditional model. If we fail to reject such a null hypothesis, we find no evidence in support of the Lucas Critique. Interestingly however, in this case [as pointed out by Favero and Hendry (1990) in a survey of related work] the power of such a test can be established, and if it is high, we have strong evidence against the Lucas Critique.

The analysis of the previous section can be interpreted in an expectations framework. If μ_t^x represents an expectation formed about x_t, then (9) allows for the usual possibility that $\delta \neq \beta$ (assuming that δ is constant) as well as covering situations when δ is not constant and β varies with changes in the expectations process.

More generally, consider a model in which $\mathscr{F}_t$ denotes information available at the start of planning period t, μ_t^y is the planned value of y_t, and the plan depends on the expectation held about $\{x_{t+i}; i = 1, \ldots, N\}$. Thus, in place of (3), we have the distributed lead model:

$$\mu_t^y = \beta(\lambda_{2t})\mathrm{E}[x_t | \mathscr{F}_t] + \rho_1 \mathrm{E}[x_{t+1} | \mathscr{F}_t]$$

$$+ \cdots + \rho_N \mathrm{E}[x_{t+N} | \mathscr{F}_t] + z_t'\gamma, \tag{3*}$$

where we have taken N to be finite and will focus on the case $N = 1$ as this highlights the main analytical issues. The potential variation of ρ_i with respect to λ_{2t} would lead to extended versions of the tests described below.

To develop a maintained hypothesis, a model of x_t is postulated where

$$x_{t+1} = \mathrm{E}[x_{t+1} | \mathscr{F}_t] + v_{t+1} = \boldsymbol{\varphi}_t' \boldsymbol{Z}_t + v_{t+1}, \tag{10}$$

when $Z_t \in \mathscr{F}_t$, and φ_t potentially varies over time. Substituting (10) into (3′) for $N = 1$, and the result into (2), yields the appropriate generalization of (4):

$$y_t | x_t, \mathscr{F}_t \sim \mathrm{N}[\beta(\lambda_{2t})x_t + z_t'\gamma + \{\delta_t - \beta(\lambda_{2t})\}(x_t - \mu_t^x) + \rho_1 \varphi_t' Z_t, \omega]. \tag{11}$$

Clearly, x_t is not weakly exogenous for β even if $\delta_t = \beta(\lambda_{2t})$ since the parameter φ_t appears directly in the conditional density but must be estimated from the marginal density of x_t. The presence of the future expectation introduces a possibly time-varying function of the current information set into the regression. Such a regression model will typically appear to be nonconstant unless φ and Z have very simple forms, and will not be invariant to the process generating x_t which is incorporated in φ_t and Z_t.

To formulate a general alternative to superexogeneity in this type of rational expectations model, we allow for failures of invariance, and substitute (7) into (11) to get

$$\begin{aligned} y_t | x_t, \mathscr{F}_t \sim \mathrm{N}[x_t\beta_0 + z_t'\gamma + (\delta_t - \beta_0)(x_t - \mu_t^x) + \beta_1(\mu_t^x)^2 + \beta_2\sigma_t^{xx} \\ + \beta_3\mu_t^x\sigma_t^{xx} + \rho_1\varphi_t' Z_t, \omega]. \end{aligned} \tag{12}$$

Hence, (5) will be the estimated model if $\delta_t = \beta_0$, $\beta_1 = \beta_2 = \beta_3 = 0$, and $\rho_1 = 0$. Then tests of these restrictions will comprise a superexogeneity test in the expectations context. These assumptions guarantee the weak exogeneity of x_t for β_0 and the invariance of the conditional model to variations in the generating process of x_t. Of course, to carry out this test in practice (as discussed in the next section), it is necessary to specify and estimate parameters such as φ_t as well as μ_t^x.

Superexogeneity can hold only if there are no future expectations in the behavioral model. However, it may still be of interest to test models in which x_t is not weakly exogenous for β for failures of invariance. By replacing $\varphi_t' Z_t$ in (11) with x_{t+1} and assuming that $\delta_t = \beta(\lambda_{2t})$, possibly because $x_t \in \mathscr{F}_t$, the model has the familiar errors-in-variables form of rational expectations model popularized by McCallum [1976]:

$$y_t = \beta(\lambda_{2t})x_t + \rho_1 x_{t+1} + \gamma' z_t + \varepsilon_t - \beta(\lambda_{2t})v_{t+1},$$

which requires estimation with instruments (Z_t, z_t) in order to achieve consistency even when $\beta(\lambda_{2t}) = \beta$. To test for invariance of β, (7) above can still be used but only after estimating with instrumental variables rather than by least squares, as in the last example of the previous section.

The ability of these tests to detect failures of superexogeneity, particularly through the expectations mechanism, is intimately connected to the complexities of the marginal process of the conditioning variable. For example, if $\boldsymbol{\varphi}_t = \boldsymbol{\varphi}$ and $\boldsymbol{Z}_t = z_t$, then the restriction $\rho_1 = 0$ is not testable. Thus, the power of the testing procedures can be ascertained by examining the marginal processes for changing parameters and additional causal variables.

The argument of this section can be generalized to multiple future leads. Formulating the multi-step predictors as $\mathrm{E}_t x_{t+i} = \boldsymbol{\varphi}'_{it}\boldsymbol{Z}_t$ yields the conditional model:

$$y_t \mid x_t, \mathscr{F}_t \sim \mathrm{N}\Bigg[x_t\beta_0 + z_t'\gamma + (\delta_t - \beta_0)(x_t - \mu_t^x) + \beta_1(\mu_t^x)^2 + \beta_2\sigma_t^{xx} + \beta_3\mu_t^x\sigma_t^{xx} + \sum_{i=1}^{N} \rho_i\boldsymbol{\varphi}'_{it}\boldsymbol{Z}_t, \omega \Bigg], \tag{13}$$

where each of the expectations terms could be tested separately, except that for most forcing variables x only a small subset would be linearly independent.

The most widely-used model of forward expectations is the present discounted value (PDV) model which has been used to explain stock prices as a function of dividends, long rates as a function of short rates, and consumption as a function of income. In each case, it is typically assumed that a conditional model is misspecified, and it is assumed that the model is invariant with constant discount factors. We consider in somewhat more detail the testing of the present discounted value model.

Consider the example where y_t is an end-of-period stock price, x_t is its dividend, and as before, $\mathscr{F}_t$ is the sigma field including past values of y and x as well as current values of valid conditioning variables. The PDV model is

$$\mathrm{E}[y_t \mid \mathscr{F}_{t+1}] = y_t = \beta \sum_{i=1}^{\infty} \theta^i \mathrm{E}[x_{t+i} \mid \mathscr{F}_{t+1}], \qquad \theta = 1/(1+r),$$

which can be rewritten as the backward-looking model

$$\mathrm{E}[y_t + \beta x_t \mid \mathscr{F}_t] = (1+r)y_{t-1}$$

or

$$\mu_t^y = -\beta\mu_t^x + (1+r)y_{t-1},$$

where it is desired to ascertain whether $\beta = 1$. This is now in the form of (3) and can be examined for superexogeneity of x_t. From (9), $z_t = y_{t-1}$, $\gamma = (1+r)$, and the test would require $\beta_1 = \beta_2 = \beta_3 = 0, \delta_t = \beta_0$. There is nothing in the theory which would suggest that the weak exogeneity component of this hypothesis would be true. Hence, we expect to reject the null hypothesis of superexogeneity as we expect that this truely is a forward-looking system. Estimates of β_0 above 1 indicate excess sensitivity of prices to dividends, but these are easily confused by the natural simultaneity between y and x.

To test whether β is invariant to λ_{2t} and equal to 1 without assuming weak exogeneity, one must either use instrumental variables or estimate the reduced form. In this context, one can test interesting hypotheses about the expectations mechanism. For example, suppose the PDV model is only geometric for higher leads, so that

$$y_t = \beta \sum_{i=1}^{\infty} \theta^i \mathrm{E}[x_{t+i} | \mathscr{F}_{t+1}] + \sum_{i=1}^{I} \psi_i \mathrm{E}[x_{t+i} | \mathscr{F}_{t+1}],$$

and we wish to test that $\psi_i = 0$, $\forall i$. Rewriting this model gives

$$y_t = (1+r)y_{t-1} + [-\beta - (1+r)\psi_1]x_t + \sum_{i=1}^{I} \{\psi_i \mathrm{E}[x_{t+i} | \mathscr{F}_{t+1}]$$

$$- (1+r)\psi_{i+1} \mathrm{E}[x_{t+i} | \mathscr{F}_t]\},$$

where $\psi_{I+1} \equiv 0$, and computing the reduced form by taking expectations conditional on $\mathscr{F}_t$ gives

$$\mathrm{E}[y_t | \mathscr{F}_t] = (1+r)y_{t-1} + [-\beta - (1+r)\psi_1]\mu_t^x$$

$$+ \sum_{i=1}^{I} \{\psi_i - (1+r)\psi_{i+1}\} \mathrm{E}[x_{t+i} | \mathscr{F}_t],$$

which is of the form of (13) and can be used to test hypotheses on the expectations mechanism by testing for invariance in the relation between y and x when the process of x changes.

5. Tests for superexogeneity

A variety of tests for superexogeneity will be proposed; tests for invariance in models without weakly exogenous conditioning variables only need modification along the lines sketched above. In each case, the null hypothesis will be that x_t is superexogenous for β in the regression:

$$y_t = x_t \beta + z_t' \gamma + \varepsilon_t. \tag{5}^*$$

Different tests will be sensitive to different alternatives, and we separately consider worlds where Σ_t is constant and where it varies over time (whether under the null or alternative).

5.1. Σ_t is constant

Assuming temporarily that $\beta_i = 0$ ($i = 1, 2, 3$), this case is the classical issue of testing for weak exogeneity. Wu (1973) and Hausman (1978) proposed tests and Engle (1982a) established their optimality through showing them to be Lagrange multiplier tests. To construct a test, μ_t^x must be parameterized. Suppose there is a set of instruments $\boldsymbol{Z}_t$, including z_t, which describe the mean of x through

$$x_t = \boldsymbol{Z}_t' \boldsymbol{\pi}_x + \eta_t. \tag{14}$$

The construction of $\boldsymbol{Z}_t$ is assumed to allow for and define regime shifts. Some of the variables in $\boldsymbol{Z}_t$ may be lagged x's and y's or dichotomous variables interacting with other observable variables. This specification, therefore, gives wide scope to specifying changes in policy regime, expectations formation, or states of nature.

If $\mathrm{E}[\varepsilon_t \eta_t] \neq 0$, then there will be simultaneous equations bias. The LM test for weak exogeneity is to test the estimate of $\{\eta_t\}$ given from (14) as $x_t - \boldsymbol{Z}_t' \hat{\boldsymbol{\pi}}_x = x_t - \hat{x}_t = \hat{\eta}_t$ for its presence in the model (5). Notice that in eq. (4) with δ and β constant, their difference is zero if η_t has a zero coefficient. The t-statistic on $\hat{\eta}_t$ is a simple and intuitive form of the test. Since $x_t = \hat{x}_t + \hat{\eta}_t$, an equivalent form would be the t-statistic on $\hat{x}_t$. If $\hat{\eta}_t$ belongs in the regression but is excluded, then other diagnostic tests may have some power. In particular, parameter constancy tests have been used in this case, although their power is crucially dependent on changes having occurred in the underlying data process. Indeed, constancy tests are an example of one choice of $\boldsymbol{Z}$ corresponding to using (0, 1) dummy variables. However, for an H-period constancy test, H dummies are required and the lack of a constructive alternative either for the break period or for the cause of the break suggests that they will lack power relative to a directed test of the form proposed here. Moreover, a failure to reject may simply reflect a constant within-sample process for x and so throw no light on invariance.

In the common case where one is unwilling or unable to specify the entire set $\boldsymbol{Z}$, it may still be possible to perform the test. Partition $\boldsymbol{Z}$ into $(z, \boldsymbol{Z}_1, \boldsymbol{Z}_2)$ where $\boldsymbol{Z}_1$ and $\boldsymbol{Z}_2$ are excluded from (5) on *a priori* grounds (rather than merely because of insignificant coefficients in pre-tests) and $\boldsymbol{Z}_1$ is observed. $\boldsymbol{Z}_1$ might be dummies for shifts in regimes which, under superexogeneity, ought not to enter (5). A test of superexogeneity would then be whether $\boldsymbol{Z}_1$ enters (5). A test for any linear combination of $\boldsymbol{Z}_1$ would also be such a test and, from (4), the linear combination most closely approximating μ_t^x should be most powerful. The test can be constructed as before: regress x_t on $(z_t, \boldsymbol{Z}_{1t})$ and test $\hat{x}_t$ (or equivalently $\hat{\eta}_t$) in eq. (5).

An alternative interpretation of these tests is insightful. Under the null that $\mathrm{E}[\varepsilon_t \eta_t] = 0$, the restricted reduced form for y_t derived from the two-equation system (5) and (14) is

$$y_t = \beta \boldsymbol{\pi}_x' \boldsymbol{Z}_t + \gamma' z_t + \varepsilon_t + \beta \eta_t = \boldsymbol{\pi}_y' \boldsymbol{Z}_t + u_t \quad \text{(say)}. \tag{15}$$

The unrestricted reduced form is simply the regression of y_t on $\boldsymbol{Z}_t$. Thus, one can equivalently test whether the former encompasses the latter [for discussions of encompassing, see Hendry and Richard (1982), Mizon (1984), Mizon and Richard (1986), and Hendry and Richard (1989)]. One degree of freedom tests [see Cox (1961), Pesaran (1974), and Ericsson (1983)] evaluate variance encompassing which corresponds to testing: $\sigma_u^2 = \sigma_\varepsilon^2 + \beta^2 \sigma_\eta^2$, where σ_u^2 is estimated from the unrestricted reduced form. Providing that $\mathrm{E}[\varepsilon_t \eta_t] = 0$, *variance encompassing* will indeed ensue. Tests based on this approach are reported in Favero (1989). *Parameter encompassing* tests will have l degrees of freedom for l restrictions on $\boldsymbol{\pi}_y$, and as shown in Mizon (1984), these tests are equivalent to the test of independence of instruments and errors in Sargan (1964).

If we now allow $\beta_i \neq 0$ $(i = 1, 2, 3)$ but retain $\boldsymbol{\Sigma}_t = \boldsymbol{\Sigma}$, then we have $\hat{x}_t$ entering the regression through its square as well as its level. Thus one would perform an F-test on $\hat{x}_t$ and $\hat{x}_t^2$. If we further allow the possibility of future expectations, then we must form multi-step predictors of future x. Letting $\hat{x}_{ti}$ be the estimated forecast of x_{t+i} using information available at time t, the test procedure would perform a joint test on $\hat{x}_t, \hat{x}_t^2, \hat{x}_{t1}, \ldots, \hat{x}_{tN}$.

5.2. $\boldsymbol{\Sigma}_t$ *varying*

In this more general case, the coefficient of x_t is potentially varying as shown in (9). Note that if, for example, σ_t^{xx} is constant within regimes, but not between, then one might find weak exogeneity within each regime but no superexogeneity. Charemza and Kiraly (1986) suggest recursive analogues of the tests in section 5.1 above to detect such cases.

To develop formal testing procedures, expand $\sigma_t^{yx}/\sigma_t^{xx} = \delta_t = \delta_0 + \delta_1 \sigma_t^{xx}$ so that (9) can be written in testable form as:

$$\begin{aligned} y_t = {} & x_t \beta_0 + z_t' \gamma + (\delta_0 - \beta_0)\hat{\eta}_t + \delta_1 \sigma_t^{xx} \hat{\eta}_t + \beta_1 \hat{x}_t^2 \\ & + \beta_2 \sigma_t^{xx} + \beta_3 \hat{x}_t \sigma_t^{xx} + \varepsilon_t, \end{aligned} \tag{16}$$

where again $x_t = \boldsymbol{Z}_t'\hat{\boldsymbol{\pi}}_x + \hat{\eta}_t = \hat{x}_t + \hat{\eta}_t$. If we suppose that σ_t^{xx} has distinct values over different but clearly defined regimes, then the three separate hypotheses described in section 2 are susceptible to test:

(a) *Weak exogeneity of x_t for β_0*. This entails a zero effect from $\hat{\eta}_t$ as in section 5.1.
(b) *Constancy of δ entails $\delta_1 = 0$*. Given (15), constancy tests should be conducted for the coefficients of $\hat{\eta}_t$ and $\hat{x}_t$, or equivalently for x_t and $\hat{\eta}_t$.
(c) *Invariance of β*. This entails that $\beta_i = 0$ $(i = 1, 2, 3)$ in (15) taking σ_t^{xx} to be given by disjoint values.

Depending on how rich a maintained hypothesis is desired, the statistic could jointly test for the significance of all the terms involving δ and β. Notice, however, that $z_t'\gamma$ will *not* have changing coefficients in the present formulation.

From the reduced form regression $x_t = \boldsymbol{Z}_t'\boldsymbol{\pi}_x + \eta_t$, it is possible to draw inferences about σ_t^{xx}. A heteroscedasticity function could be fit which specifies $\mathrm{var}(\eta_t \mid \mathscr{F}_t) = \boldsymbol{Z}_t'\boldsymbol{\kappa}$. Even simpler might be the ARCH model of Engle (1982b) which allows a flexible form of conditional heteroscedasticity:

$$\mathrm{var}(\eta_t \mid \mathscr{F}_t) = \boldsymbol{\kappa}_0 + \Sigma \boldsymbol{\kappa}_i \eta_{t-i}^2, \tag{17}$$

so that $\boldsymbol{Z}_t$ corresponds to lagged values of η_t^2. Estimation of (14) with (17) will improve the efficiency of the estimates in (14) and provide an estimated series $\hat{\sigma}_t^{xx}$ to use with $\hat{x}_t$ in testing (16). Moreover, both $\{\hat{\sigma}_t^{xx}\}$ and $\{\hat{\eta}_t^2 - \hat{\sigma}_t^{xx}\}$ could be included to check on the impact of systematic versus unanticipated error variance changes.

Throughout this section it has been assumed that $\omega_t = \sigma_t^{yy} - (\sigma_t^{xy})^2/\sigma_t^{xx}$ is constant under both the null and the alternative. While this is surely true under the null, there may be power lost by failing to define an expression for ω_t as a function of σ_t^{xx}. There does not seem to be any natural such expression, however, and so we maintain $\omega_t = \omega$ throughout, letting σ_t^{yy} adjust. This assumption is itself testable empirically, or might suggest the use of robust standard errors for the various tests as in White (1980).

6. An empirical illustration

The example selected relates to testing the superexogeneity of the parameters of the money demand function in Hendry (1988). This is a particularly appropriate example because considerable debate has addressed parameter constancy and exogeneity assumptions in this context. For example, a long sequence of predictive failures in U.S. M1 is documented by Judd and Scadding (1982) and reviewed and interpreted by Baba, Hendry, and Starr (1992). In addition, the exogeneity assumptions implicit in money demand models have been criticized by Cooley and Leroy (1981) and Hendry and Ericsson (1991) among others. The issue is that if the money stock M is taken as a policy variable whose value depends only on past information, then at least one of the interest rate, R, real income, Y, or the price level, P, must be endogenous. If the last of these is selected as endogenous, models of real money should be formulated with P as the dependent variable potentially allowing conditioning on M. Instead, some studies condition on P, taking M as the dependent variable [see, e.g., H.M. Treasury (1980)]. Since both M and P cannot be weakly exogenous for the parameters of the money demand function, and in most cases, both cannot have constant parameterizations, both conditional models cannot be invariant to the monetary rule. In short, the exogeneity assumptions are usually very important.

The measure of money is quarterly M1 in the U.K. over the period 1963–1982, the precise dates depending on the lag lengths of the various models. The income measure is Total Final Expenditure, the price measure is the implicit deflator of TFE, and the interest rate is the three-month Local Authority bill rate. Lowercase letters denote logarithms of capitals. The original model takes the form:[2]

$$\begin{aligned}\Delta[m-p]_t = & \underset{(0.144)}{-0.854}\Delta p_t \underset{(0.001)}{-0.007}R_t \underset{(0.091)}{-0.354}\Delta[m-p]_{t-1} \\ & \underset{(0.104)}{+0.280}\Delta y_{t-1} \underset{(0.011)}{-0.105}ECM_{t-2} \underset{(0.006)}{+0.031},\end{aligned} \tag{18}$$

$T = 1965(2)–1982(4)$, $R^2 = 0.732$, $\sigma = 1.31\%$, $DW = 1.90$, Mean $= -0.00293$, $SD = 0.0244$, $\chi^2(2) = 1.87$, AR1–5$F[5,60] = 0.35$, ARCH 4$F[4,57] = 0.02$, $Xi^2\, F[10,54] = 1.62$, RESET $F[1,64] = 0.43$,

In (18), Δ denotes a first difference, $ECM_t = (m - p - y)_t$, and R^2, σ, DW, Mean, and SD, respectively, denote the squared multiple correlation coefficient, the residual standard deviation, the Durbin–Watson statistic, the mean quarterly growth rate of real money, and its unconditional standard deviation. Also, AR 1–5 F-test (with degrees of freedom shown) is a Lagrange multiplier test for residual autocorrelation; ARCH 4 F-test checks for residual heteroscedasticity of the ARCH form; Xi^2, again in F-form, is the test suggested by White (1980); $\chi^2(2)$ is the Jarque–Bera (1980) normality statistic; and the RESET test is due to Ramsey (1969). None of these diagnostic checks is significant, nor is *any*

[2]All of the calculations were undertaken using PC-GIVE 6.0 [see Hendry (1989)].

recursively computed 'break point' Chow test, so (18) also appears to be constant over the sample period. Note that (.) and [.] beneath coefficients denote conventional and heteroskedastic consistent standard errors, respectively.

There have been several potential regime shifts over the sample period, including the Competition and Credit Control Regulations of 1971, the shift from fixed to floating exchange rates in that year, the oil crisis in late 1973, the change in the Value Added Tax rate from 8% to 15% in 1979, and the tight monetary policy introduced by the Thatcher Government after 1979. Dummy variables were created for step changes [e.g., *D73(4)* denoting starting year and quarter] or impulses [e.g., *D*69(1/2)* denoting the + 1, − 1 periods]. Level and interactive combinations of these dummy variables were tried for the impact of these potential shift events in the marginal models for p_t and R_t and any first round significant effects were retained. The resulting models took the form:

$$\begin{aligned}\Delta p_t = {} & \underset{(0.004)}{0.009}D79(3)_t - \underset{(0.008)}{0.051}D73(4)_t - \underset{(0.008)}{0.055}D73(4)_t p_{t-1} \\ & + \underset{(0.108)}{0.488}D73(4)_t \Delta p_{t-1} + \underset{(0.005)}{0.030}p_{t-1} + \underset{(0.006)}{0.022}\Delta D79(3) + \underset{(0.008)}{0.056},\end{aligned} \tag{19}$$

$T = 1963(3)$–$1982(4)$, $R^2 = 0.870$, $\sigma = 0.549\%$, $DW = 2.01$, Mean $= 0.0234$, $SD = 0.0146$, $\chi^2(2) = 0.43$, AR 1–5 $F[5, 66] = 0.91$, ARCH4 $F[4, 63] = 0.20$, Xi^2 $F[9, 61] = 0.94$, RESET $F[1, 70] = 0.03$,

and

$$\begin{aligned}\Delta R_t = {} & -\underset{(0.060)}{0.202}R_{t-1} + \underset{(0.106)}{0.408}\Delta R_{t-1} + \underset{(1.19)}{3.07}\Delta D73(4)_t \\ & + \underset{(0.444)}{1.496} + \underset{(0.424)}{0.897}D73(4)_{t-1},\end{aligned} \tag{20}$$

$T = 1963(3)$–$1982(4)$, $R^2 = 0.277$, $\sigma = 1.148$, $DW = 2.07$, Mean $= 0.0727$, $SD = 1.314$, $\chi^2(2) = 3.12$, AR1–5 $F[5, 68] = 0.90$, ARCH 4 $F[4, 65] = 3.63$, Xi^2 $F[6, 66] = 1.46$, RESET $F[1, 72] = 0.50$.

In (19) and (20), the 'oil crisis' dummy is significant in several forms, but the dummy for the VAT increase to 15% in 1979 is important only in the Δp_t equation. Both equations pass the diagnostic checks for residual autocorrelation, residual heteroscedasticity, normality, and the RESET test. However, residual ARCH is significant for ΔR_t, and is modelled below. Overall, (19) and (20) seem reasonable marginal models for the analogues of μ_t^x, especially since the standard errors (σ) are about $\frac{1}{2}$% and one point, respectively, and there is strong evidence of structural breaks in the linear projections alone. However, the presence of many dummies precluded recursive constancy tests.

Eq. (20) suggests that R_t is integrated of order zero denoted I(0) [see Granger (1986) and Engle and Granger (1987)] and that the oil crisis raised interest rates considerably, both in the short run and the long run (by about 4.5 points). However, it is much harder to interpret (19): certainly, the VAT change raised inflation substantially, but whether or not the oil crisis did so depends on both the prevailing price level and inflation rate. Despite this problem of interpretation, both equations strongly reflect important regime effects compared to conventional autoregressive representations, which is the key requirement for the illustrations of this section.

From these two models, $\hat{\eta}(\Delta p)_t$, $\hat{\eta}(R)_t$, $\hat{x}(\Delta p)_t$, and $\hat{x}(R)_t$ were calculated and then measures of σ_t^{xx} were constructed: for Δp_t, a three-period moving average of $\hat{\eta}_t^2(\Delta p)$ was tried despite the insignificance of the ARCH test [denoted $V(\Delta p)$]; for R, a two-period ARCH error denoted $A(R)$ was estimated [$F[2,69] = 6.1$ in (20)] as was the deviation of $\hat{\eta}_t^2(R)$ from $A(R)$ [denoted $Dev(R)$]. All of these constructed variables were then included in the money demand function, which had conditioned on both Δp_t and R_t; in addition, the subgroups of variables which were most promising for rejecting invariance were included. However, no combination proved significant, and (21) reports the estimates when all five test constructs were added:

$$\begin{aligned}\Delta[m-p]_t = & -\underset{[0.155]}{0.754}\Delta p_t - \underset{[0.001]}{0.006}R_t - \underset{[0.105]}{0.259}\Delta[m-p]_{t-1} \\ & + \underset{[0.137]}{0.310}\Delta y_{t-1} - \underset{[0.014]}{0.100}ECM_{t-2} + \underset{[0.008]}{0.026} - \underset{[0.283]}{0.392}\hat{\eta}(\Delta p)_t \\ & + \underset{[70.4]}{43.1}V(\Delta p)_t - \underset{[0.002]}{0.002}\hat{\eta}(R)_t - \underset{[0.002]}{0.003}A(R)_t - \underset{[0.001]}{0.001}Dev(R)_t,\end{aligned} \tag{21}$$

$T = 1965(2)\text{–}1982(4)$, $R^2 = 0.750$, $\sigma = 1.32\%$, $DW = 1.95$, Mean $= -0.00293$, $SD = 0.0244$, $\chi^2(2) = 0.50$, AR 1–5 $F[5, 56] = 0.60$, ARCH4 $F[4, 53] = 0.89$, $Xi^2\,F[16, 44] = 0.57$, RESET $F[1, 60] = 0.09$, F-test [for adding $\hat{\eta}(\Delta p)_t$, $V(\Delta p)_t$, $\hat{\eta}(R)_t$, $A(R)_t$, $Dev(R)_t$]: $F[5, 60] = 0.85$.

Thus, the evidence is consistent with the super exogeneity of Δp_t and R_t in the original M1 model.

A natural issue is the power of this class of tests, and to investigate that, two variants of the basic model were evaluated. The first omitted Δp_t and hence imposed price homogeneity at all lags rather than just in the long run, and the second replaced Δp_t by Δp_{t-1} as a solved form, with the possible interpretation of an extrapolative predictor of inflation. In both cases, strong rejection was obtained as seen below:

$$\begin{aligned}
\Delta[m-p]_t = {} & \underset{[0.001]}{-0.006}R_t \underset{[0.111]}{-0.056}\Delta[m-p]_{t-1} \underset{[0.151]}{+0.269}\Delta y_{t-1} \underset{[0.008]}{+0.022} \\
& \underset{[0.015]}{-0.080}ECM_{t-2} \underset{[0.391]}{-1.157}\hat{\eta}(\Delta p)_t \underset{[73.6]}{-77.0}V(\Delta p)_t \\
& \underset{[0.002]}{-0.002}\hat{\eta}(R)_t \underset{[0.002]}{-0.005}A(R)_t \underset{[0.001]}{-0.001}Dev(R)_t,
\end{aligned} \tag{22}$$

$R^2 = 0.679$, $\sigma = 1.48\%$, $DW = 1.97$, $\chi^2(2) = 0.50$, RESET $F[1, 60] = 0.09$, AR 1–5 $F[5, 56] = 0.60$, ARCH 4 $F[4, 53] = 0.89$, $Xi^2\,F[16, 44] = 0.57$, F-test [for adding $\hat{\eta}(\Delta p)_t$, $V(\Delta p)_t$, $\hat{\eta}(R)_t$, $A(R)_t$, $Dev(R)_t$]: $F[5, 61] = 3.47$,

and

$$\begin{aligned}
\Delta[m-p]_t = {} & \underset{[0.001]}{-0.007}R_t \underset{[0.128]}{-2.95}\Delta[m-p]_{t-1} \underset{[0.144]}{+0.236}\Delta y_{t-1} \\
& \underset{[0.014]}{-0.103}ECM_{t-2} \underset{[0.008]}{+0.026} \underset{[0.317]}{-1.093}\hat{\eta}(\Delta p)_t \underset{[67.1]}{-17.1}V(\Delta p)_t \\
& \underset{[0.002]}{-0.002}\hat{\eta}(R)_t \underset{[0.002]}{-0.003}A(R)_t \underset{[0.001]}{-0.001}Dev(R)_t \underset{[0.145]}{-0.648}\Delta p_{t-1},
\end{aligned} \tag{23}$$

$R^2 = 0.735$, $\sigma = 1.36\%$, $DW = 1.87$, $\chi^2(2) = 2.34$, RESET $F[1,59] = 0.07$, AR 1–5 $F[5,55] = 0.34$, ARCH 4 $F[4,52] = 0.30$, $Xi^2\,F[18,41] = 0.59$, F-test [for adding $\hat{\eta}(\Delta p)_t$, $V(\Delta p)_t$, $\hat{\eta}(R)_t$, $A(R)_t$, $Dev(R)_t$]: $F[5,60] = 2.99$.

In particular, both the weak exogeneity component of inflation and the anticipated error variance of interest rates are significant in the two misspecified models, even though (23) without the test variables is almost as well-fitting as the selected equation. Thus, the test procedure has some power and the nonrejection of (21) is consistent with its previously established constancy over more than five years after first selection.

Interestingly, (23) without the test variables is constant over the sample period as judged by recursive 'break point' Chow tests: there is no split sample point commencing in 1965(2) and ending in 1982(4) where rejection would have been obtained at even the 5% level for a single test. Thus, this misspecification would not have been detected by any of the other tests reported here.

Finally, we consider the 'inversion' of (19) to determine inflation as a function of an assumed exogenous money stock and the arguments of the money demand function. If the money demand equation is invariant and constant, then the inverted equation will not be. The Chow statistics for a structural break are computed for each point in the sample period. Although the critical values of the maxima of these are the subject of continuing research, the contrast with earlier models is marked since constancy would have been rejected at the 0.1% level by any investigator testing almost anywhere within sample, despite the constancy of (19).

7. Conclusion

The main result of this paper is that hypotheses of invariance and superexogeneity of regression parameters can be tested directly, and the proposed tests seem to have power to detect incorrect assertions. In an example where a previously established model was tested, nonrejection was obtained, whereas rejection resulted from two misspecified variants of that model. Further examples are provided in Favero (1989) who uses superexogeneity, cointegration, and encompassing tests to distinguish between approximate and exact models of the expectations theory of the term structure of interest rates. Recursive analogues of the tests in section 3 would facilitate joint investigation of constancy and invariance, especially in the context of expectations-based equations.

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