

## Problem Set 4: Forecasting and Non-stationarity

### 1) Forecasting using more variables

1. Estimate an AR(1) model for  $\log(\text{CO}_2)$  up until 1990 and use a 1-step forecast to generate forecasts for  $\log(\text{CO}_2)$  from 1991-2010. What is the root-mean-squared forecast error?
2. Use the Kuznets curve model you have estimated previously (see below in (2)) and forecast  $\log(\text{CO}_2)$  from 1991 until 2010 using 1-step forecasts and contemporaneous RGDP values. Comment on the results (e.g. RMSE) and compare them to the simple AR(1) model estimates. The model to be estimated:

$$\log(\text{CO}_2)_t = \beta_1 + \beta_2 \log(\text{RGDP})_t + \beta_3 \log(\text{RGDP})_t^2 + \epsilon_t \quad (1)$$

Note: these are not “true” forecasts as we are predicting values in time  $t$  using RGDP information from the same time  $t$ .

3. The partial auto-correlation function suggests that we should include one lag of  $\log(\text{CO}_2)$  into the model. Estimate the model by adding a lagged value of  $\log(\text{CO}_2)$  and compare the forecasts over the same time period to the above models:

$$\log(\text{CO}_2)_t = \beta_1 + \beta_2 \log(\text{CO}_2)_{t-1} + \beta_3 \log(\text{RGDP})_t + \beta_4 \log(\text{RGDP})_t^2 + \epsilon_t \quad (2)$$

### 2) Linear Trend

Think back on the graphs of the series we are modelling from problem set 1. As visual inspection shows, the  $\log(\text{CO}_2 \text{ Emissions per capita})$ ,  $\log(\text{RGDP})$ , and  $(\log(\text{RGDP}))^2$  are slightly trending. We now investigate these underlying trends.

1. Investigate the presence of a linear trend in the series: Augment the previous model with a linear time trend. Is the coefficient on the linear trend variable significant? What does the result tell you about whether a trend should be included or not?

### 4) Big Picture

These questions have no clear right answers – use your interpretation and creativity!

1. Overall, given the evidence from your estimated models, what do you conclude on the the environmental Kuznets curve for Japan?
2. How could the models you have estimated be improved further?
3. How would you structure an analysis of the environmental Kuznets curve in a short paper, from presenting the data to analysis and conclusions? (providing a short outline is sufficient, do not write an essay.)