# Problem Set 3: Time Series Properties and Forecasting

So far the estimated models of the Japanese Kuznets curve have not accounted for potential time dependence or lagged effects. The following exercises ask you to investigate the time series properties and use your models to forecast.

## 1) Time series Properties of the Data

Now we turn to the time series properties of the data:

1. Plot the Auto-correlation and Partial auto-correlation functions of log(CO2) and log(RGDP) and comment on the plots. What lag length would you suggest is optimal?

2. Estimate an auto-regressive model with 1-lag (AR-1 models) for log(CO2), report the results in equation format and comment on the output.

$$\log(\text{CO2})_t = \alpha_0 + \alpha_1 \log(\text{CO2})_{t-1} + \epsilon_t \quad (1)$$

## 2) Auto-regressive Model Forecasts

Investigate the forecasting performance of the models you have estimated so far. For the current scenario, imagine it is the year 1990 and you are interested in forecasts of GPD per capita and CO2 emissions per capita.

1. Estimate AR(1) models for log(RGDP) and log(CO2) up until 1990 and compute the static long-run solution (you can set the sample length up until 1990 when being asked to choose the estimation sample in PcGive).

   - Use a 1-step forecast to generate forecasts from 1991-2010, plot the forecast graphs, comment on the performance. What is the root-mean-squared forecast error?
   - Use a dynamic forecast to generate forecasts from 1991-2040 (50 years), comment on the outcome. How do the results relate to the long-run solution? Do the results seem sensible to you?

## 3) Static Model Forecasts

1. Use the model you have estimated previously (2) and forecast log(CO2) from 1991 until 2010 using 1-step forecasts and contemporaneous RGDP values. Comment on the results and compare them to the simple AR(1) model estimates. The model to be estimated:

$$\log(\text{CO2})_t = \beta_1 + \beta_2 \log(\text{RGDP})_t + \beta_3 \log(\text{RGDP})_t^2 + \epsilon_t \quad (2)$$

   over a sample period up until 1991.